(Max, +) Optimization Model for Scheduling Operations in a Flow Network with Preventive Maintenance Tasks

Karla Quintero, Eric Niel, José Aguilar, and Laurent Piétrac

***Abstract*—The aim of this work is to propose a (max, +) optimization model for scheduling transfer operations on a flow network within a given maintenance framework. The case study involves the scheduling of oil batch transfer operations in coordination with valve maintenance activities in an oil-exporting seaport. The optimum schedule is determined through an intuitive, and synthetized mathematical model based on (max,+) algebra with the objective of minimizing financial penalties. Real operational constraints and goals in the seaport are modeled with data from an oil seaport in Venezuela. Results show the optimum schedule obtained from a concise and relatively simple optimization model which is the main contribution of this work.**

***Index Terms*—system modeling, (max,+) theory, flow networks, schedule optimization.**

I. Introduction

THE following work proposes a (max, +) optimization model for operations' scheduling on a flow network, using as a case study a seaport for oil export. A pipeline network is the core of the physical system supporting several oil transfer and maintenance operations; therefore, in a given time frame, conflict phenomena due to resource assignment naturally arise. The contribution of this work lies on the intuitive and concise mathematical modeling of the optimization problem through (max,+) algebra which, to our knowledge, has not been applied to this type of system. We formulate a schedule optimization model through an industrial application of (max, +) algebra with data from an oil seaport in Venezuela. Moreover, the results are extendable to applications to flow networks of different nature.

Other common approaches dealing with conflict resolution on resource allocation include Petri Nets, specifically event graphs, where conflicts are previously solved through a routing policy, i.e. a criterion that enables the choice of one transition among a group of conflicting transitions demanding to be fired; [1], [2], and [3] can be consulted for an overview on common routing policies. Other approaches deal with conflict directly within the framework of the resolution algorithm; for instance, [4] implements an ant colony optimization algorithm in which conflict is modeled as a probabilistic choice rule depending on the pheromone trail and a heuristic function. Conversely, we neither assume a pre-established routing policy nor a dependency on the resolution algorithm. The focus of this work lies on building a generic algebraic model which, based on (max, +) constraints, determines the optimum operations' schedule that minimizes the total cost of penalties in the system for a given time horizon. Furthermore, if absolute priorities were to be known between every given pair of conflicting operations, then the resulting model would be a (max,+)-linear system of the type $X = AX$ (see [5]) to which control theory for linear systems could be applied.

In classic formulations for schedule optimization problems, dependencies are expressed as less intuitive and concise constraints. For instance, in [6], an optimization model for flow-shop scheduling with setup times is formulated as sets of recursive constraints expressing the underlying dependency between completion times for jobs on machines. In [7] and [8], for instance, classic resource conflict constraints are expressed through decision variables imposing a precedence and therefore forcing one machine operation to depend on the completion time of a conflicting one. These same principles constitute the base of the (max,+) approach but instead, with the proper algebraic structure (i.e. fundamental mathematical operators, decision variables based on the zero and/or identity element, and mathematical properties such as commutativity, idempotency, and distributivity, among others) formulations can be more intuitively constructed and additional and more intricate phenomena (such as maintenance activities in this work) can be easily integrated. Furthermore, depending on the system's properties and the optimization goal, (max,+)-linear systems (i.e. $X = AX$) can be obtained as aforementioned.

Section II presents some preliminary notions on (max, +) algebra. Section III covers the system description, related work, and some operational aspects for operations' scheduling in a Venezuelan seaport. Resource allocation notions are described in section IV, and section V presents the proposed (max, +) optimization model with the respective results in section VI.

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II. (MAX,+) ALGEBRA OVERVIEW

(max, +) algebra is defined as a mathematical structure denoted as $\mathbb{R}_{max}$, constituted by the set $\mathbb{R} \bigcup \{-\infty\}$ and two binary operations $\oplus$ and $\otimes$, which correspond to $maximization$ and $addition$, respectively. This algebraic structure is an idempotent commutative semifield. As [5] states, a semifield $\mathcal{K}$ is a set endowed with two generic operations $\oplus$ and $\otimes$. Operation $\oplus$ is associative (e.g. $a \oplus (b \oplus c) = (a \oplus b) \oplus c$), commutative (e.g. $a \oplus b = b \oplus a$) and has the zero element ε (e.g. $a \oplus \varepsilon = a$); and operation $\otimes$ is distributive with respect to $\oplus$ (e.g. $a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c)$) and its identity element e satisfies $\varepsilon \otimes e = e \otimes \varepsilon = \varepsilon$. The semifield is idempotent if the first operation is idempotent (i.e. $a \oplus a = a, \forall a \in \mathcal{K}$). Moreover, in a semifield, operation $\otimes$ must be invertible (e.g. in (max,+) algebra: if $2 \otimes 3 = 5$ then $2 = 5 \oslash 3$ or in conventional notation: if $2 + 3 = 5$ then $2 = 5 - 3$). In (max,+) algebra, the zero element is $\varepsilon = -\infty$ and the identity element is $e = 0$. Some basic examples on the use of operators are $2 \oplus 3 = 3, 2 \oplus 2 = 2, 2 \oplus \varepsilon = 2, 2 \oplus e = 2, 2 \otimes 3 = 5, 2 \otimes 2 = 4, 2 \otimes \varepsilon = \varepsilon, 2 \otimes e = 2$.

(max, +) models aim at describing the system's main properties through two basic mathematical operations: maximization and addition. As for which systems are to be modeled with this tool, those exhibiting synchronization phenomena as their main feature are the best direct candidates. However, research in this field continues to explore further possibilities. In this work, (max, +) algebra is applied to a system in which resource allocation conflicts constitute the main characteristic. (max,+) theory is a research field that has caught the attention of the scientific community for its intuitive modeling potential of discrete event system's phenomena that would usually involve more intricate mathematical models. For further information on (max, +) algebra for production chains and transportation networks [9] can be consulted. [5] can be consulted for (max, +)-linear system theory, [10] for (max, +) theory applied to traffic control, [11] for an application to production scheduling in manufacturing systems, and [12] for maintenance modeling for a helicopter. Moreover, considerable effort has been dedicated to exploiting the potential of (max, +) automata; see [13], [14], and [15] for developments in this field. To our knowledge, no work has yet been developed to optimize pipeline networks' scheduling while integrating maintenance operations based on a (max, +) approach.

III. SYSTEM DESCRIPTION

A. Oil Transfer Operations

An intricate pipeline network links a set of tanks storing oil to be exported and a set of loading arms placed at the docks of the seaport. Loading arms are connected to tankers (i.e. the clients) that receive the oil and transport it to different countries. An oil transfer is carried out by selecting an alignment (i.e. a path) of pipelines linking the two elements of interest and enabling oil flow by opening the valves in the alignment and closing all adjacent valves in order to isolate it to avoid oil mixture[1]. We consider that oil flow from the tank to the loading arm is enabled by gravity, as it is the case in some Venezuelan oil seaports. This work assumes that the proper alignment has been previously selected to satisfy each request and the addressed problem is the scheduling of requests while respecting a predefined maintenance schedule in order to minimize penalties in the system. Previous work related to the case study includes some approaches on alignment selection; [16] can be consulted for alignment selection minimizing interferences with envisaged operations in the network, and [17] for alignment selection maximizing operative capacity while minimizing failure risk on valves. Maintenance operations are to be executed on valves and, in order to do so, all adjacent valves (also called 'isolating valves') must be closed. Fig. 1(a) depicts an example of a simplified oil seaport, and Fig. 1(b) shows its model as an undirected graph in which arcs represent the valves and the nodes represent pipeline segments. A schedule is determined in terms of client requirements and on valve[2] availability in order to enable alignments.

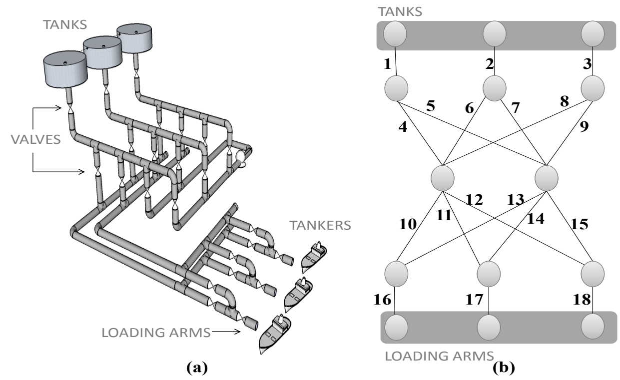

Fig. 1. Oil seaport example (a) and its undirected graph model (b)

B. Maintenance Operations

Scheduling of maintenance operations implies an entire research field. Typical aspects to consider are device reliability; repair, replacement and inspection costs; condition monitoring costs; as well as potential costs for not applying the proper maintenance operations, among others. In this work, a maintenance schedule is assumed to have been properly generated by the specialized maintenance personnel, and we study the scheduling of oil transfer operations in order to minimize penalties while fully respecting pre-established maintenance operations.

C. Penalty Management on a Seaport for Oil Export[3]

Each oil transfer operation has an associated deadline which, if violated, implies monetary penalties. Hence, the

[1]Even though one case could correspond to the mixture of two identical oil types, in this research, oil mixture is not allowed in any scenario since sharing an alignment section by two transfer operations could result in lower product flow rate and aspects such as pumping power and pipeline dimensions would have to be considered and are not the focus of this work.

[2]Pumps are not modeled since in many of these oil seaports oil flows by gravity, and maintenance on pipeline segments is not part of this research.

[3]These operational aspects were gathered through direct collaboration with PDVSA and one of its oil seaports in Venezuela. Most of the aspects discussed in this research still hold in the case of seaports for oil import and even for flow networks of a different nature.

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seaport aims at minimizing the *Total Cost due to Penalties* (TCP) for a time frame with *nc* clients. For each client, a negotiation takes place, typically a month and a half before the transfer operation. The client imposes (under certain conditions not relevant to this work) for a specific tanker, the penalty (in thousands of dollars per hour) to be paid by the seaport in the case of a seaport-caused delay. At the same time, the seaport imposes a time window of three days for the tanker's arrival. From the moment of the tanker's arrival within this time window, the maximum service time is 36 hours for loading and 4 hours for paperwork. Since the focus of this paper is on transfer operations, we concentrate solely on the maximum loading time of 36 hours as the deadline. From that point on, if a delay is caused by the seaport, every extra hour of loading results in a penalty for the seaport. Conversely, if the delay is caused by the tanker, then the client incurs in penalties for dock over-occupation. Client-paid penalties do not represent in any way an optimization objective, i.e. they are unforeseen events which the seaport does not aim at maximizing through operation scheduling. If the tanker arrives after its time window, the seaport does not incur in any penalties for the tanker's waiting time. No further information has been gathered concerning other arrival scenarios. For model validation purposes, we assume that if the tanker arrives before its time window, the 36 hours of service are counted from the starting point of the authorized time window. Since deadlines depend on arrival dates and interruptions that can cause service delays, each time an event occurs in the network the schedule must be recalculated in order to adapt to up-to-date operational conditions.

IV. Resource Allocation on an Oil Seaport

Here we describe possible conflicts involving valve allocation. Namely, conflicts between oil transfer operations and between transfer operations and maintenance tasks. These notions apply to any flow network managing different products.

A. Conflicts between Different Oil Transfer Operations

Definition 1: Two or more alignments (for oil transfers) are in conflict if they share at least one valve and if either the valve requires different states for different alignments or it requires being open for more than one alignment.

Fig. 2(a) shows two disjoint alignments to satisfy requests R_1 and R_3. Solid lines illustrate the valves to open and dotted lines (of the same color) the valves to close in order to isolate the alignment; e.g.: to enable R_1 valves 1, 4, 10, and 16 must open and valves 5, 6, 8, 12, 11, and 13 must close. In Fig. 2(a), no conflict arises since common resources (valves 5, 8, 12, and 13) are all valves to be closed, therefore they can enable both transfer operations simultaneously. In Fig. 2(b), another request is added and conflicts arise for valves 10 and 16, since they should open for 2 transfer operations (therefore, mixing 2 types if oil), and for valves 4 and 6, since their required commutations are different (which is physically impossible). Therefore, R_1 and R_2 cannot be processed simultaneously. Naturally, it is of paramount importance to serve as many clients as possible in the shortest amount of time, this translates into simultaneous execution of transfer operations whenever possible with the goal of minimizing the TCP.

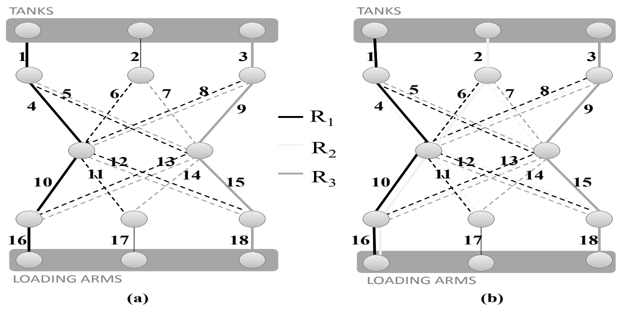

Fig. 2. Non-conflicting and conflicting alignments for oil transfer operations

B. Conflicts between Oil Transfer Operations and Maintenance Operations on Valves

Definition 2: A valve can enable oil flow in an alignment or it can isolate the alignment, but it cannot simultaneously be subject to maintenance.

In Fig. 2(b), if valve 6 were to be maintained, it would generate a conflict with R_1 (since valve 6 is required closed) and of R_2 (since valve 6 is required open). Since for a valve to be maintained, it must be isolated from the rest of the network by closing all adjacent valves, we must also ensure there is no conflict between the isolating valves for maintenance and the oil transfer operations requiring their use as an open valve. This has not been addressed explicitly, however, the proposed model manages these conflicts implicitly as stated in Definition 3.

Definition 3: For a valve, the conflict between its request as an isolating valve for maintenance and as an open valve for oil transfer in an alignment will always generate a conflict between the valve in maintenance and an isolating valve for the alignment in question.

The proposed (max, +) model represents implicitly this conflict type because each commutation (open/close) is modeled for each valve in order to enable oil flow in an alignment. For example, in Fig. 2(b), if valve 7 is in maintenance, then valves 2 and 6 (i.e. input valves) and valves 5, 9, 13, 14, and 15 (i.e. output valves) should all be isolating valves and, for instance, valve 2 cannot be used at the same time to satisfy R_2. However, this conflict (isolating valve for maintenance/open valve for transfer) is implicitly solved by the other arising conflict (isolating valve for request/valve in maintenance, previously dealt with in Definition 2), for valve 7 since it should be in maintenance but it should also be closed for R_2.

V. Proposed (MAX,+) Optimization Model

The model is based on (max, +) algebra which allows intuitively to model conflicts in resource allocation and all remaining constraints for the optimization problem.
In the following, let $\mathcal{O}$ be the set of all possible commutations on valves in order to satisfy a set of nc requests so that $\forall ikl : ikl \in \mathcal{O}$, where k is a valve to commutate to a state l (open/closed = $1/0$) to satisfy request i. Analogously,

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let $\mathcal{M}$ be the set of all maintenance activities previously scheduled so that $\forall hk : hk \in \mathcal{M}$ where h is the number of the maintenance operation to be executed on valve k (e.g. $hk = 13$ states that a maintenance operation denoted as 1 is to be executed on valve 3, whereas $hk = 23$ states that a second maintenance operation is executed on valve 3). The set of isolating valves for a maintenance operation hk is denoted ISO_{hk}. The first constraint of the optimization model corresponds to (1) in conventional algebra, (2) being its equivalent in (max, +) notation. In the following, only (max, +) notation will be used. This constraint determines the start date (x_{ikl}), also called *dater* in the (max, +) framework, for a commutation to satisfy an oil transfer operation. Variables are: x_{ikl} as aforementioned; xp_{hk}: dater for a maintenance operation h on valve k; $x_{i'kl'}$: dater for a conflicting request i' requesting valve k; $V_{ikl,hk}$: binary decision variable which ultimately solves the precedence between oil transfer operation i and the maintenance operation; $V_{ikl,i'kl'}$: analogously, defines the precedence between two conflicting requests i and i'; and u_i: tanker's arrival date for request i. $ztp_{hk}, zp_{i'}$, and $zc_{i'}$ represent, respectively, the possible unforeseen delays in the maintenance operation, in the service of a client due to technical difficulties in the terminal and in the service of a client due to difficulties within the tanker.
Parameters include: t, tp_{hk}, and $p_{i'}$ which are respectively the start date of the scheduling time horizon and the nominal durations for the maintenance activity and the oil transfer operation.

$$x_{ikl} = max\big(t; u_i; max_{hk}(xp_{hk} + tp_{hk} + ztp_{hk} + V_{ikl,hk}); max_{i'}(x_{i'kl'} + p_{i'} + zp_{i'} + zc_{i'} + V_{ikl,i'kl'})\big), \forall\ ikl, i'kl' \in \mathcal{O} | [i \neq i' \wedge (l \neq l' \vee l = l' = 1)], \forall hk \in \mathcal{M} \quad (1)$$

$$x_{ikl} = t \oplus u_i \oplus \big(\oplus_{hk}(xp_{hk} \otimes tp_{hk} \otimes ztp_{hk} \otimes V_{ikl,hk})\big) \oplus \big(\oplus_{i'}(x_{i'kl'} \otimes p_{i'} \otimes zp_{i'} \otimes zc_{i'} \otimes V_{ikl,i'kl'})\big) \forall ikl, i'kl' \in \mathcal{O} | [i \neq i' \wedge (l \neq l' \vee l = l' = 1)], \forall hk \in \mathcal{M} \quad (2)$$

Equation (2) states that the dater for a commutation to satisfy a request depends on the start date of the scheduling horizon, the arrival date of the tanker, the maximum completion time of all conflicting maintenance operations which precede request i, and the maximum completion time of all conflicting oil transfer operations preceding request i. All decision variables are binary, taking the values $e(zero)$ or ε. For instantiation purposes, values are e or B so that B is a very large negative real number. Moreover, each decision variable has a complementary one (e.g. if $V_{ikl,i'kl'} = e$ then $V_{i'kl',ikl} = B$ or vice versa). In (2), if $V_{ikl,hk} = B$ then the entire term $xp_{hk} \otimes tp_{hk} \otimes ztp_{hk} \otimes V_{ikl,hk}$ is negligible, which implies that the completion time of maintenance operation hk does not determine x_{ikl}; this indicates that maintenance on valve k is executed after request i. Conversely, if $V_{ikl,hk} = e$, then the same term represents the completion time of the maintenance activity which means it precedes the oil transfer.

Analogously to (2), on (3) the dater for a maintenance activity is calculated. Although the start dates of maintenance activities have already been fixed, (3) restrains the accepted values for the decision variables of conflicting operations. In (3), the result is the maximum of three terms: the first is the fixed date for the maintenance activity which forces the equality, the second one models the conflict with other requests and the third term models the conflict between the maintenance on k and the possible maintenance on the isolating valves for k. Equations (2) and (3) interact through the values of the complementary decision variables. To solve a conflict between commutation $ikl = 241$ for a request $i = 2$ and a maintenance activity $hk = 14$ (both requesting valve 4) the resolution technique would assign values to the decision variables which would generate the daters, thus, if $V_{241,14} = B$ then $V_{14,241} = e$ which implies that in (2) x_{241} does not depend on that maintenance's completion time and on (3) xp_{14} does depend on the completion time of the oil transfer, therefore, the transfer precedes the maintenance operation. Conversely, if the values of the decision variables were inverted, then the maintenance operation would precede the oil transfer. As for which scenario is preferable, the decision is made based on the resulting TCP.

$$xp_{hk} = xp_{hk} \ \oplus\ \big(\oplus_i(x_{ikl} \otimes p_i \otimes zp_i \otimes zc_i \otimes V_{hk,ikl})\big) \oplus \big(\oplus_{h'k'}(xp_{h'k'} \otimes tp_{h'k'} \otimes ztp_{h'k'} \otimes V_{hk,h'k'})\big) \ \forall\ i|ikl \in \mathcal{O},\ \forall\ hk \in \mathcal{M},\ \forall\ h'k' \in ISO_{hk} \quad (3)$$

In (4), for all valves in an alignment that satisfies a request i, all daters are equal. Hence, commutation times are negligible compared to the duration of the oil transfer. Since pipelines are always full of oil, the client starts receiving the oil batch 'almost' immediately[4].

$$x_{ikl} = x_{ik'l'} \qquad \forall i \in nc, \forall ikl, ik'l' \in \mathcal{O} \quad (4)$$

$$V_{ikl,i'kl'} \otimes V_{i'kl',ikl} = B \quad (5)$$

$$V_{ikl,i'kl'} \oplus V_{i'kl',ikl} = e \quad (6)$$

Equations (5) and (6), restrict the values of conflicting oil transfers to $e(zero)$ and B, whereas (7) and (8) do the same for conflicting maintenance and transfer operations. Finally, (9) and (10) restrict the values for conflicting operations for maintenance and isolation of valves to be maintained. To simplify notation we omit indices on (5-10) without loosing clarity of the model.

$$V_{ikl,hk} \otimes V_{hk,ikl} = B \quad (7) \qquad V_{ikl,hk} \oplus V_{hk,ikl} = e \quad (8)$$

$$V_{hk,h'k'} \otimes V_{h'k',hk} = B \quad (9)$$

$$V_{hk,h'k'} \oplus V_{h'k',hk} = e \quad (10)$$

$$D_i = \begin{cases} u_i \otimes 36 & \forall\ i|u_i \in tw_i \\ x_{ikl} \otimes 36 & \forall\ i|u_i > utw_i \\ ltw_i \otimes 36 & \forall\ i|u_i < ltw_i \end{cases} \quad (11)$$

In (11) the deadline D_i for a request i is modeled. $tw_i = [ltw_i, utw_i]$ is the arrival time window of three days. If the tanker arrives within this time window, its deadline is 36 hours after its arrival, if it arrives afterwards the seaport does not incur in any penalties (as it has been confirmed by

[4]This assumption has been kept from previous work. It implies that the amount of oil stored in pipelines is also negligible compared to the amount of oil requested by the client and therefore not relevant if it has the same specifications as the requested batch.

the seaport) for the waiting time to be docked. We assume (for validation purposes) its deadline as the start date of the transfer operation plus 36 hours. Finally, if the tanker arrives before its time window, we assume (since no further information has been gathered) that the deadline is the lower bound of the time window plus the mandatory 36 hours.

$$dpr_i = (x_{ikl} \otimes p_i \otimes zp_i \otimes zc_i \oslash D_i) \oplus e \qquad \forall i | ikl \in \mathcal{O} \quad (12)$$

Hypothesis 1: The dock over-occupation penalty per hour per client (paid by each client) is considered equal to the penalty per hour for that same client paid by the seaport in case of delay caused by the seaport[5].

The *delay per request* (dpr) is determined in (12). For each request, the difference is calculated between the completion time of the request (including the possible delays caused by the seaport and/or the client) and its deadline. No further information has been gathered for scenarios where both the client and the seaport incur in penalties. For validation purposes, we rely on Hypothesis 1 and, thereby, if both parties incur in delays of the same length, no penalty is paid. However, if the delays are not equal, the party with the greatest delay pays the difference between both delays. Equation (13) models the *penalized delay for the seaport* (pds) per request; i.e. the time interval (hours) for which the seaport will pay the respective penalties. Here, if the tanker's waiting time (modeled as zu_i) plus all loading interruptions caused by the seaport (zp_i) is greater than the interruptions caused by the client, then the seaport incurs in a potential penalty. This penalty is the minimum between the difference of delays ($zu_i + zp_i$ and zc_i) and the dpr_i (which is the actual time exceeded since the deadline). This minimization, which is translated in (13) into a maximization in (max, +) algebra, aims at penalizing only the seaport delay that actually surpasses the established deadline. If the delay caused by the tanker is greater than or equal to the delay caused by the seaport, then the seaport does not incur in penalties.

$$pds_i = \begin{cases} \oslash\left[(\oslash zu_i \oslash zp_i \otimes zc_i) \oplus (\oslash dpr_i)\right] \\ \qquad\qquad\qquad \forall (zu_i \otimes zp_i) > zc_i \\ e \qquad\qquad\quad otherwise \end{cases} \quad (13)$$

$$Min\ TCP = \otimes_i \left(\otimes_{n=1}^{pds_i} c_i \right) \qquad \forall i \in nc \quad (14)$$

Equation (14) represents the objective function of the optimization problem. It computes the *Total Cost due to Penalties* (TCP) for all requests in the time horizon. It is the (max, +) algebra representation for the sum of the products of each penalized delay (in hours) and its penalty (in \$/hour).

VI. Results

The model is instantiated with a simplified topology (as in Fig. 1) in order to visually grasp the complexity of decision making for scheduling several operations with potential conflicts, and how the problem becomes more complex as the network's size increases. The instantiation is done using the optimization tool LINGO (see [18]) where several algorithms can be chosen to solve optimization problems. Here, we use the global solver which guarantees finding the global optimum; the solver repeatedly tries values for decision variables (which generates values for all daters) until the objective can no longer be improved while respecting all constraints. The instance includes seven oil transfer requests to be scheduled (denoted as $R_p,\ p = 1, \ldots, 7$) and maintenance activities on valves 13 and 15 at the dates of 100 and 130 hours and with durations of 12 and 10 hours, respectively. The alignments for such requests are specified in Fig. 3 (only open valves for each alignment are depicted for easier comprehension), as well as the valves to be maintained (where, analogously, isolating valves are not depicted). This instance covers all types of possible conflicts and input data is presented on Table I.

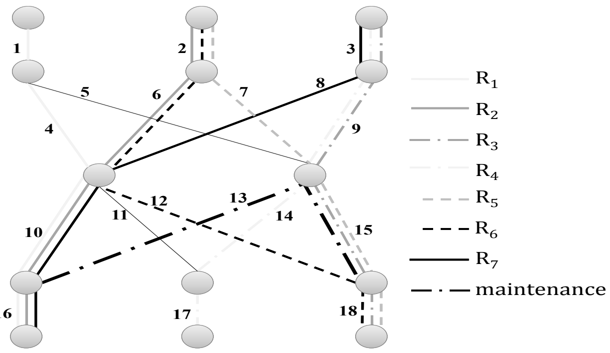

Fig. 3. Operations to be scheduled

TABLE I
Input data for oil transfer operations

Request	Processing Time (hours)	Penalty (\$/hour)	Time Window for arrival (days)
R_1	20	4000	[4,6]
R_2	25	2500	[2,4]
R_3	20	3000	[2,4]
R_4	15	2500	[1,3]
R_5	20	2500	[1,3]
R_6	15	3000	[2,4]
R_7	10	2000	[3,5]

Since a time window is authorized for tanker arrival, a reference schedule can be obtained by assuming certain arrival dates (within or outside the time windows). For validation purposes, it is assumed that all tankers, except the one for R_2, arrive within their time windows, at the last hour of the last day. It is also assumed that the tanker for R_2 arrives after its time window at 10 a.m. of day 5 (one standard time scale in hours is used to illustrate results). Also, no interruption that could cause additional delay on service is considered which translates into $zp_i = zc_i = ztp_i = e$. All of these values should be adjusted dynamically (which implies schedule recalculation) as more information is gathered by the seaport in terms of actual expected arrival dates, and service interruptions, among others. The resulting optimum schedule that generates the minimum TCP of \$137000 is shown in Fig. 4. For illustration purposes, the chosen instance deliberately forces the seaport to pay penalties given the tight constraints in terms of number of clients, tanker arrival dates and processing times. Considering the relatively simple topology,

[5]this is assumed for validation purposes only, and can be adjusted according to each flow network

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it can be verified manually that no other schedule generates a lower TCP. Naturally, not one conflicting operation overlaps with another and all scheduled maintenance tasks are fully respected. Namely, the requests for which the seaport incurs in penalties are: R_3 with a delay of 29 hours and R_4 with a delay of 20 hours, which multiplied by their respective costs yields the resulting TCP. Notice that the seaport does not incur in any penalties for R_2 since the tanker arrives after its time window.

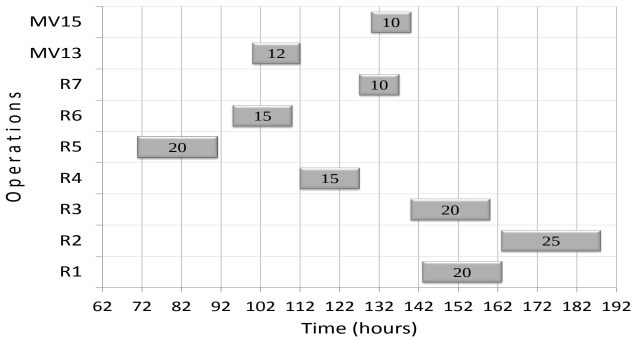

Fig. 4. Optimal schedule

We emphasize that the main objective has been to minimize the TCP according to real needs and operational data supplied by the collaborating industrial company PDVSA. However, further work for this type of network or for flow networks of different nature could be the minimization of the TCP within a just-in-time production framework (which would generate the latest dates at which service can be started for each client) or, conversely, within an earliest production context. Moreover, relaxation of maintenance dates could be adressed. The importance of these results lies in finding the desired solution through an approach that has not yet been adressed, to our knowledge, to solve this type of flow network optimization problem, i.e. an algebraic approach that allowed us to concisely formulate all optimization needs using nothing but addition and maximization. Moreover, if absolute priorities were known for clients, then the system's optimization model would be a (max,+)-linear model of the form $X = AX$ (where X corresponds to the vector of commutation daters, for both transfer and maintenance operations, A represents all dependencies between daters and AX is the application of the matrix (max,+) product). Further work could exploit this system representation in order to apply classic control theory for linear systems.

VII. Conclusion

The proposed (max, +) model optimizes oil transfer operations while ensuring reliability of the system through predefined maintenance tasks on valves. The advantage of this algebraic discrete event approach is that it provides, exclusively through operators of maximization and addition, all necessary elements to represent the proposed optimization needs and constraints in a clear and concise manner. The goal has been to exploit this formal and mathematical modeling approach and set the framework for more complex (max, +) models for the case study. More specifically, further work should consider maintenance relaxation through appropriate time windows and potential additional criteria: such as maintenance schedule optimization (through maintenance costs and reliability on valves), and alignment selection optimization for each request. Furthermore, the application of (max,+) automata is envisaged for scheduling through supervisory control of the system while imposing the latest maintenance dates for devices.

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