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Book of abstracts

M. Błaszak, *Separability and R-separability of Stäckel matrices.*

We demonstrate the way of separable quantization of wide class of Stäckel systems with quadratic in momenta separation relations. In general, separable quantization is not related with any Stäckel metric. Quantizing the Hamiltonian with respect to its Stäckel metric we get a generalized R-separability as the quantum Hamiltonian contains a quantum correction term. We also search for classical R-separability when quantum correction vanishes.

Jan L. Cieśliński, *Isothermic surfaces and their generalizations. New developments.*

Geometrically, isothermic surfaces are characterized by curvature lines admitting conformal parameterization. The corresponding Gauss-Codazzi equations are integrable and both matrices of the Lax pair are linear functions of the spectral parameter [1]. A natural hypothesis that this spectral parameter is a group parameter has been recently confirmed (a joint work with Artur Kobus). In my talk I consider a class of spectral problems linear in the spectral parameter with matrices replaced by elements of a general Clifford algebra [2]. The continuous case is integrable and the Darboux transformation is known in a general case [3]. The discretization preserving geometric features of the continuous case has been also proposed [4] but its integrability is an open problem except the simplest case of isothermic surfaces in E^3 [5]. In my talk I show some new results on the Darboux transformation for the discrete case. The main idea is to treat both cases, continuous and discrete, simultaneously (time scales approach, compare [6]). The form of the Darboux matrix turns out to be exactly the same in both cases but most of reductions are more difficult to be preserved in the discrete case.

[1] J.Cieśliński, P.Goldstein, A.Sym: Isothermic surfaces in E^3 as soliton surfaces, Phys. Lett. A 205 (1995) 37-43.

[2] J.L.Cieśliński: A class of linear spectral problems in Clifford algebras, Phys. Lett. A 267 (2000) 251-255.

[3] W.Biernacki, J.L.Cieśliński: A compact form of the Darboux-Bäcklund transformation for some spectral problems in Clifford
algebras, Phys. Lett. A 288 (2001) 167-172.

[4] J.L.Cieśliński: Discretization of multidimensional submanifolds associated with Spin-valued spectra problems, J.Math.Sci. 149
(2008) 1032-1038.

[5] J.Cieśliński: The Bäcklund transformation for discrete isothermic surfaces, [in:] Symmetries and Integrability of difference
equations, pp. 109-121, edited by P.A.Clarkson and F.W.Nijhoff, Cambridge University Press 1999.

[6] J.L.Cieśliński, T.Nikiciuk, K.Waśkiewicz, The sine-Gordon equation on time scales, J. Math. Anal. Appl. 423 (2015) 1219-1230.

A. Doliwa, *Generalized quasi-symmetric functions and Hopf algebras of trees.*

Theory of symmetric functions is by now well established subject with numerous applications in algebraic topology, combinatorics, representation theory and geometry. It plays also an important role in free-fermion approach to the KP hierarchy, and in quantum integrability. The quasi-symmetric functions are extensions of the symmetric ones, and are becoming of comparable importance. I would like to present new generalization (inspired by the Connes-Kreimer approach to renormalization in QFT) of the quasi-symmetric functions together with geometric description of their Hopf algebra structure. I will also describe the generalized quasi-symmetric functions in more standard way in terms of series of bounded degree, but in partially commuting variables.

C. Gonera, *Non-spherically symmetric superintegrable potentials.*

Some superintegrable models with Hamiltonians separable in spherical coordinates will be presented and discussed.

P. R. Gordoa, *On Painlevé hierarchies related to the Boussinesq hierarchy.*

We consider a system of equations defined using the Hamiltonian operator of the Boussinesq hierarchy, as well as two successive modifications thereof. We are able to reduce the order of the three systems and also to give Bäcklund transformations between the integrated systems. We can also construct auto-Bäcklund transformations for the two modified systems. For a particular case, we identify generalized fourth Painlevé hierarchies. As a consequence we derive auto-Bäcklund transformations for these fourth Painlevé hierarchies as well as Bäcklund transformations between our Painlevé hierarchies. Moreover, the results on reduction of order provide a reduction of order of the scaling similarity reduction of the Boussinesq hierarchy.

Y. Grigoriev, *On the separation of variables for the two-dimensional integrable system with velocity-dependent potential.*

We are working within a scope of applying transformations of bi-Hamiltonian structure to classify known integrable systems and find new ones. In the earlier work, the equations analogous to Bäcklund transformations were shown to be connected with the form of Lax matrix and can be explicitly derived from it. An algorithmic construction of the auto Bäcklund transformations of Hamilton-Jacobi equations is discussed and possible applications of this algorithm to finding new integrable system with integrals of motion of higher order in momenta are presented.

P. Kassotakis, *2^n-rational maps.*

We present a natural extension of the notion of nondegenerate rational maps (quadrirational maps) to arbitrary dimensions. We refer to these maps as 2^n-rational maps. We introduce a rich family of 2^n-rational maps. These maps by construction are involutions and highly symmetric in the sense that the maps and their companion maps have the same functional form. For $n = 2$ (quadrirational case) we reconstruct the F and H list of Yang-Baxter maps. For $n = 3$ (octorational case) by a singular reduction we obtain a map associated to discrete A-KP.

A. Lelito, O. Morozov, *Group-invariant solutions of Gibbons-Tsarev equation.*
The Gibbons-Tsarev equation is of the form:

$$u_{yy} = (u_y + y)\, u_{xx} - u_x\, u_{xy} - 2, \tag{1}$$

where $u = u(x, y)$ and subscripts denote partial derivatives. It has been widely known since it arised in [1] as a special case of a reduction of Benney's moment equations [2]. This origin of Gibbons-Tsarev equation connects it directly with a model (also presented in the above-mentioned Benney's work), which is meant to describe behaviour of long waves on shallow, inviscid and incompressible fluid.

Using well-known tools provided by the theory of Lie groups, we find solutions of Gibbons-Tsarev equation that are invariant with respect to its symmetries. In order to examine the problem of finding group-invariant solutions in as much systematic way as possible, we applied the method based on searching for an optimal system of one-dimensional subalgebras of the symmetry algebra of Eq.(1). Hence, every other group-invariant solution can be derived from one of the solutions we obtained.

Then we use the obtained solutions to construct explicit expressions for two-component reductions of Benney's moments equations, to get solutions of Pavlov's equation, and to find integrable reductions of the Ferapontov–Huard–Zhang system, which describes implicit two-phase solutions of the dispersionless Kadomtsev-Petviashvili equation.

[1] J. Gibbons, S.P. Tsarev, *Reductions of the Benney equations*, Phys. Lett. **A 211**, 1996, 19–24

[2] D.J. Benney, *Some properties of long nonlinear waves*, Stud. App. Math. **52**, 1973

A. J. Maciejewski, J. Gill, *Dynamics of chains in external fields.*
We discusses the dynamics of systems of point masses joined by massless rigid roods in the field of a potential force. General form of equations of motion for such systems is obtained. We investigate integrability of these equations in a case when the chain moves in constant and linear field of forces. Moreover, the dynamics of a linear chain of mass points moving around a central body in an orbit is analysed. The non-integrability of the chain of three masses moving in circular Kepler orbit around a central body is proven. This was achieved thanks to an analysis of variational equations along two particular solutions and an investigation of their differential Galois groups.

K. Marciniak, *Classical and quantum superintegrability of Stäckel systems.*
In this talk I will discuss maximal superintegrability of both classical and quantum Stäckel systems. I will demonstrate a sufficient condition for a fl‡at or constant curvature Stäckel system to be maximally superintegrable. Further, I will demonstrate a sufficient condition for a Stäckel transform to preserve maximal superintegrability and further apply this condition to the considered class of Stäckel systems, which yields new maximally superintegrable systems as conformal deformations of the original systems. Further, I show how to perform the procedure of minimal quantization to considered systems in order to produce quantum superintegrable and quantum separable systems.

M. Nieszporski, *From 2^n-rational maps to integrable systems.*

A link beetwen involutive rational maps and difference integrable systems allows us to unify difference integrable equations and corresspondences. I will show that the link can serve as classification tool for the systems. I will present the menagerie of integrable systems that revealed while we were investigating the link, including correspondences and difference equations on four-point, six-point and seven-point stencils.

M. Przybylska, *Integrability conditions for homogeneous deformations of radial potentials.*

They are well known differential Galois integrability obstructions for Hamiltonian systems with n degrees of freedom governed by Hamiltonians $H = T+V$, where T is the standard kinetic energy and $V = V(q)$ is a homogeneous function of coordinates. These obstructions have the form of restrictions on eigenvalues of Hessian matrix $V(d)$ calculated at points d satisfying $V'(d) = d$ called Darboux points and they demonstrated many times its usefulness. Very recently one made an effort to extend such kind of obstructions to the case when potential V is a finite sum of homogeneous components. The most complete results were obtained for potentials V of the form $V = V_k+V_K$, where V_k and V_K are homogeneous functions of integer degrees k and K, $K > k$, respectively, provided V_k and V_K have a common Darboux point. Condition of possessing a common Darboux points is satisfied by an arbitrary homogeneous potential V_k of degree k with at least one Darboux point and a radial potential $r^{2s} = (q^T q)^s$ where $2s$ is an integer. We will present necessary integrability obstructions of natural Hamiltonian systems with potentials $V = V_k + ar^{2s}$. We consider in details special cases of homogeneous perturbations of the harmonic oscillator when $s = 1$ and the Kepler problem when $s = -1/2$. Examples of obtained integrable systems will be shown.

A. Pickering, *Integrable systems and properties of Painlevé hierarchies.*

We discuss the connections between the properties of completely integrable PDE or lattice hierarchies and those of related Painlevé hierarchies. We see that bi Hamiltonian structures, Miura maps and the factorization of Hamiltonian operators can be used to show that solutions of members of one Painlevé hierarchy also give solutions of members of a different Painlevé hierarchy, or to derive Bäcklund and auto-Bäcklund transformations for Painlevé hierarchies.

S. Rauch-Wojciechowski, *Global dynamics of a rolling and sliding disc, asymptotic solutions, stability.*

The problem of a disc rolling in a plane has been treated in classical works of P.Apple, D.J.Korteweg, E.J.Routh and S.A.Chaplygin. It is described by a dynamical system of 4 equations, has 3 integrals of motion and is integrable. When sliding is allowed there are 2 more dynamical variables, equations are dissipative, non-integrable and have energy as a monotonously decreasing function of time. The key for understanding the dynamics are asymptotic solutions, their stability properties and a La Salle type theorem on asymptotic behavior of solutions. These results, together with with computer simulations of solutions starting in vicinity of the asymptotic solutions provide a basis for global understanding of what happens for different initial conditions. I shall explain formulation of the problem, present analytical results and tell how much we have learnt from numerical simulations.

A. Sergeyeyev, *On the separation of variables for the two-dimensional integrable system with velocity-dependent potential.*

We are working within a scope of applying transformations of bi-Hamiltonian structure to classify known integrable systems and find new ones. In the earlier work, the equations analogous to Bäcklund transformations were shown to be connected with the form of Lax matrix and can be explicitly derived from it. An algorithmic construction of the auto Bäcklund transformations of Hamilton-Jacobi equations is discussed and possible applications of this algorithm to finding new integrable system with integrals of motion of higher order in momenta are presented.

W. Szumiński, *Integrability and super integrability of certain homogeneous Hamiltonian systems.*

We investigate a class of natural Hamiltonian systems with two degrees of freedom. The kinetic energy depends on coordinates but the system is homogeneous. Thanks to this property it admits, in a general case, a particular solution. Using this solution we derive necessary conditions for the integrability of such systems investigating differential Galois group of variational equations. Some integrable as well as super-integrable cases are detected.

V. Yurov, *The construction of exact solutions to the Cauchy problem for the generalized hyperbolic Novikov-Veselov equation.*

We introduce a new procedure for construction of the exact solutions to Cauchy problem of the real-valued (hyperbolic) Novikov-Veselov equation. The procedure in question makes use of the well-known Airy function $Ai(\xi)$ which in turn serves as a solution to the ordinary differential equation $\frac{d^2z}{d\xi^2} = \xi z$. In addition, we demonstrate that a similar procedure can be constructed for the n-th order generalizations of the Novikov-Veselov equation, provided that one replaces the Airy function with the appropriate solution of the ordinary differential equation $\frac{d^{n-1}z}{d\xi^{n-1}} = \xi z$.