Set theory and Algebra.

3 TUE

- → Set theory
- → Relation
- → Function
- → Lattice and boolean algebra.
- → Group theory.

Set :- Unordered collection of non-repetitive element.

$A = \{1, 2, 3\}$

$B = \{1, 2, 3, 4\}$

4 WED

$C = \{1, 2, 2, 3\}$ ✗ not a set

$= \{1, 2, 3\}$.

$A = \{1, 2, 3\}$

$B = \{3, 2, 1\}$

$A \equiv B$.

$\{\ \}$ ✓

ϕ ✓ } emptyset.

$\{\phi\}$ → a set with ϕ as an element.

Notes

5 THU $A = \{1, 2, 3\}$

$2 \in A$

$3 \in A$

$\{2\} \in A$ X

$\{4\} \notin A$.

$\{\ \} = \emptyset$

$\{1\} = \lambda$

$S = \{1, \{2\}, \{3, 4\}\}$

$\{2\} \in S$, $1 \in S$, $3 \in S$ X.

Power set :-

6 FRI $A = \{1, 2, 3\}$

$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.

$n\{P(A)\} = 2^n$.

$A = \{\emptyset, \{\emptyset\}\}$

$n\{P(A)\} = 2^2 = 4$.

$A = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, 5\}$

$n(P(A)) = 2^4 = 16$

$B = \{\emptyset, \{1,2\}, \{\{1\},2\}\}$

$n(P(B)) = 2^3 = 8.$

$\{\{\}, \{\emptyset\}, \{1,2\}, \{\{1\},2\}, \{\emptyset, \{1,2\}\}, \{\{1,2\}, \{\{1\},2\}\},$
$\{\emptyset, \{\{1\},2\}\}, \{\emptyset, \{1,2\}, \{\{1\},2\}\}\}$

$A \cup B = A + B$

$A \cap B = A \cdot B$

$A^c = \bar{A}$

$\left.\begin{array}{l} A - B \\ A \cap B^C \\ A - (B \cap A) \end{array}\right\} \equiv A \cdot \bar{B}$

$A \oplus B = (A-B) \cup (B-A) \equiv A\bar{B} + \bar{A} \cdot B.$ (Symmetric Differen

ate2005 :-

$X = (A-B) - C \quad ; \quad Y = (A-C) - (B-C)$

$= A \cdot \bar{B} - C$

$= (A \cdot \bar{B}) \cdot \bar{C}$

9 MON

$Y = (A-C) - (B-C)$
$= A\bar{C} - (B\bar{C})$
$= A\bar{C}\,(\overline{B\bar{C}})$
$= A\bar{C}\,(\bar{B}+C)$
$= A\bar{B}\bar{C} + 0$
$= A\bar{B}\bar{C}$.

a) $Y \subset X$ b) $X \subset Y$ c) $X \equiv Y$ d) N.O.T

Gate2004 :-

P, Q, R is the subset of universal set U

$(P \cap Q \cap R) \cup (P^c \cap Q \cap R) \cup Q^c \cup R^c$ is equivalent

10 TUE

to

a) $P^c \cup Q^c \cup R^c$ b) $P \cup Q^c \cup R^c$ c) $Q^c \cup R^c$ d) U.

$P.Q.R + \bar{P}.Q.R + \bar{Q} + \bar{R}$
$QR + \bar{Q} + \bar{R}$
$\bar{Q} + R + \bar{R}$
$\bar{Q} + 1$
$1\ (U) \rightarrow$ Universal set.

Notes

Gate 1996 :-

$(A-B) \cup (B-A) \cup (A \cap B)$

a) $A \cup B$ b) $A^{c} \cap B'$ c) $A \cap B$ d) $A^{c} \cap B^{c}$.

$A\bar{B} + B\bar{A} + AB$

$A\bar{B} + B$

$= A + B$.

Gate 1995 :-

$S = \{\{\phi\}, 1, \{2,3\}\}$

No. of elements in Power set $P(S) = ?$

a) 2 b) 8 c) 4 d) N.O.T.

Cartesian Product of a set :-

$A = \{1,2,3\}$

$B = \{a,b\}$

$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

$B \times A = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$

$\therefore A \times B \neq B \times A$.

(Not Commutative).

13 FRI

$|A|$ - Cardinality of set = 3.
$|B| = 2$
$|A \times B| = |A| \times |B| = 6$.
$|A \times B| = |B \times A|$.

Relation :- Any Subset of $A \times B$ is a Relation

$$R_1 = \{(1,a),(1,b),(2,a),(2,b)\}.$$

$$R_1 \subseteq A \times B.$$

$A = \{1,2,3\}$
$B = \{a,b\}$

$A = \{1,2\}$
$\emptyset \subseteq A$ ✓
$P(A) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

$R = \emptyset$ void relation
or empty relation

14 SAT

$R = A \times B$ = Universal Relation

Ques :- $|A| = m$, $|B| = n$

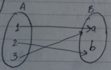
Total number of relation over $A \times B$:-

$2^{m \times n}$.

Representation of Relation :-

1) Listing method :- {(1,a),(2,b),(3,a)}
2) Set builder method :- R {(a,b) | a ≤ b} over N
3) Statement method :- R is a relation over Natural number such that a is less than or equal to b for ordered pair (a,b).

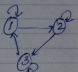
4) Arrow Diagram :-



R = {(1,a),(2,b),(3,a)}.

5) Digraph method :-

(is applicable for A×A)

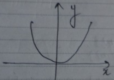


R = {(1,1),(1,2),(2,2),
(2,3),(3,3),(3,1)}

Notes

17 TUE

6) Graphical method:-



$y = x^2$

$R = (x,y) \mid y = x^2$

$\{(1,1),(2,4),\ldots\}$.

18 WED

7) Matrix Method:-

$$\begin{array}{c|ccc} & a & b & c \\ \hline 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 3 & 0 & 0 & 1 \end{array}$$

$R = \{(1,a)\ (1,b), (1,c), (2,b), (2,c), (3,c)\}$

Operation on relations:-

1) union:-

$A = \{1, 2, 3\}$

$B = \{a, b\}$

$R_1 = \{(1,a), (1,b), (2,b)\}$.

$R_2 = \{(2,b), (3,a), (3,b)\}$.

$R_1 \cup R_2 = \{(1,a), (1,b), (2,b), (3,a), (3,b)\}$.

$|R_1 \cup R_2| \leq |R_1| + |R_2|$.

2) Intersection:-

$R_1 \cap R_2 = \{(2,b)\}$.

3) Complement

$R^c = UR - R$

$= (A \times B) - R$.

4) Set Difference:-

$R_1 - R_2$

$R_1 \cap R_2^c$

21 SAT

$R_1 - R_2 = \{(1,a), (1,b)\}$.

$R_2 - R_1 = \{(3,a), (3,b)\}$.

$R_1 - R_2 \neq R_2 - R_1$

5) Symmetric difference

$R_1 \oplus R_2 = (R_1 - R_2) \cup (R_2 - R_1)$

$= \{(1,a), (1,b), (3,a), (3,b)\}$.

* $R_1 \oplus R_2 = R_2 \oplus R_1$ (Symmetric difference is commutative)

22 SUN

6) Composition operation:-

RoS :-

$R = \{(1,1), (1,2), (3,3), (2,3)\}$

$S = \{(1,1), (2,1), (3,2), (2,3)\}$.

$RoS = \{(1,1), (1,2), (2,1), (2,2), (3,3), (2,3)\}$

SoR :-

$SoR = \{(1,1), (1,3), (3,2), (2,2)\}$.

Notes

$\ast$ $RoS \neq SoR$.

$\therefore$ Composition operation is not Commutative.

Definition:-

$$RoS = \{(x,z) \mid (x,y) \in S \ \& \ (y,z) \in R\}$$

7) Inverse of Relation:-

$R = \{(1,a), (2,b), (3,a)\}$.

$R^{-1} = \{(a,1), (b,2), (a,3)\}$

$R^{-1}: \{(y,x) \mid (x,y) \in R\}$.

Types of Relation:-

$\{A \times A\}$.

I) Reflexive.
II) Symmetric
III) Transitive
IV) Irreflexive.
V) Antisymmetric
VI) Assymmetric.

Notes

25 WED

3) Reflexive Relation:-

$\forall x \quad xRx$.

i.e. $\forall x \in A \quad xRx$

$A = \{1,2,3\}$

$A \times A = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,3),(3,2),(3,3)\}$

$R = \{(1,1),(2,2),(3,3)\}$ ✓

$R_1 = \{(1,1),(2,2),(3,3),(1,2)\}$ ✓

$R_2 = \{(1,1),(2,2),(1,2)\}$ X

26 THU

max. Cardinality of Reflexive relation $= 9 = |A \times A|$

$|A| = n$.

$\therefore$ max. Cardinality of reflexive relation $= n^2$.

1) $R = \{(x,y) \mid x \parallel y\}$ — $x \parallel x$ ✓

2) $R = \{(x,y) \mid x \leq y\}$ — $x \leq x$ ✓

3) $R = \{(x,y) \mid x$ is 1 inch distance from $y\}$ — x is 1 inch from x X

4) $R = \{(x,y) \mid x \leq y\}$ — $x \leq x$ ✓

5) $R = \{(x,y) \mid x$ is brother of $y\}$ — x is a brother of x X

6) $R = \{(x,y) \mid x \mid y \rightarrow x$ divides $y\}$ — x divides x ✓

Notes

2) Symmetric Relation :-

if xRy then yRx.

$xRy \Rightarrow yRx$.

Ex:- $\{(1,2), (2,1)\}$.
$\{(1,1), (2,2), (3,3)\}$.

→ Every reflexive relation is symmetric :- False.

$\{(1,1), (2,2), (3,3), (1,2)\}$ NOT SYMMETRIC.

→ minimum reflexive relation is always symmetric :- T

1) $R: \{(x,y) \mid x \parallel y\}$. $x \parallel y \Rightarrow y \parallel x$. ✓
2) $R: \{(x,y) \mid x \leq y\}$ $x \leq y \Rightarrow y \leq x$ ✗
3) $R: \{(x,y) \mid x$ is one inch from $y\}$. ✓
4) $R: \{(x,y) \mid x \subseteq y\}$ $x \subseteq y \Rightarrow y \subseteq x$ ✗
5) $R: \{(x,y) \mid x$ is a brother of $y\}$ $x \xrightarrow{b} y \Rightarrow y \xrightarrow{b} x$ ✗.
6) $R: \{(x,y) \mid x/y\}$ $x/y \Rightarrow y/x$ ✗.

3) Transitive relation :-
if xRy & $yRz \Rightarrow xRz$.

Ex:- $\{(1,2), (2,3), (1,3)\}$.

Notes

$R = \{(1,1), (2,2), (3,3)\} \to$ Transitive.

1) $R: \{(x,y) \mid x \parallel y\}$ ✓
2) $R: \{(x,y) \mid x \perp y\}$ ✓
3) $R: \{(x,y) \mid x$ is one inch from $y\}$ ✗
4) $R: \{(x,y) \mid x \leq y\}$ ✓
5) $R: \{(x,y) \mid x$ is a brother of $y\}$ ✓
6) $R: \{(x,y) / x \mid y\}$ ✓.

i) $R = \{(x,y) \mid x+y = 10\}$.

reflexive ✗ $\quad \forall x \; xRx$ ✗, $\exists x \; xRx$ ✓

Symmetric ✓

Transitive ✗

ii) $R: x_1, y_1 \; R \; x_2, y_2$ such that $x_1 \leq x_2 \;\&\; y_1 \leq y_2$.

reflexive :- ✓

$\forall x, y \quad x_1, y_1 \; R \; x_1, y_1$

Symmetric :- ✗

$x_1, y_1 \; R \; x_2, y_2 \Rightarrow x_2, y_2 \; R \; x_1, y_1$.

Transitive ✓

4) Irreflexive relation:- $\forall x \; x \not R x$
(Not even single self loop allowed).

$R_1 : \{(1,1), (2,2), (3,3)\}$ ✗
$R_2 : \{(1,1), (1,2), (2,1)\}$ ✗
$R_3 : \{(1,2), (1,3), (2,3)\}$ ✓
$R_4 : \{(1,2), (2,1)\}$ ✓

Reflexive ⇒ Not irreflexive True
Irreflexive ⇒ Not reflexive. True
Symmetric ⇒ Irreflexive False
Not irreflexive ⇒ Reflexive. False
Not reflexive ⇒ irreflexive. False.

1) $R: \{(x,y) \mid x \parallel y\}$ ✗
2) $R: \{(x,y) \mid x \leq y\}$ ✗
3) $R: \{(x,y) \mid x$ is one inch from $y\}$ ✓
4) $R: \{(x,y) \mid x \subseteq y\}$ ✗
5) $R: \{(x,y) \mid x$ is a brother of $y\}$ ✓
6) $R: \{(x,y) \mid x \mid y\}$ ✗
7) $R: \{(x,y) \mid x+y = 10\}$ ✗

$x + x = 10$
$\Rightarrow 2x = 10$
$\Rightarrow x = 5 \rightarrow (5,5)$

∴ one self loop of $(5,5)$ is present.

1 WED

5) Antisymmetric :-

if $xRy \Rightarrow yRx$ unless $x = y$

$R_1 = \{(1,1), (2,2), (3,3)\}$ ✓
$R_2 = \{(1,1), (2,2)\}$ ✓
$R_3 = \{(1,2), (2,3)\}$ ✓
$R_4 = \{(1,2), (2,1)\}$ ✗

Symmetric ⇒ Not Antisymmetric → False.
eg :- $\{(1,1), (2,2), (3,3)\}$.

Symmetric ⇒ AntiSymmetric → False
↳ True only when self loops present.

2 THU

Not Symmetric ⇒ AntiSymmetric → True.

1) R: $\{(x,y) \mid x \parallel y\}$ ✗
2) R: $\{(x,y) \mid x \leq y\}$ ✓
3) R: $\{(x,y) \mid x$ is one inch from $y\}$ ✗
4) R: $\{(x,y) \mid x \subseteq y\}$ ✓
5) R: $\{(x,y) \mid x$ is brother of $y\}$ ✗.
6) R: $\{(x,y) \mid x / y\}$ ✓.
7) R: $\{(x,y) \mid x + y = 10\}$ ✗

6) Asymmetric :-

If $xRy \Rightarrow y \not R x$. (Also not even self-loop allowed)

Ex:- $R_1 = \{(1,1),(1,2)\}$ X

$R_2 = \{(1,2),(1,3)\}$ ✓

$R_3 = \{(1,2),(2,1),(3,3)\}$ X

$R_4 = \{(1,2),(2,1)\}$ X

1) R: $\{(x,y) \mid x \parallel y\}$ X
2) R: $\{(x,y) \mid x \le y\}$ X
3) R: {(x,y) x is 1 inch from y} X X
4) R: {(x,y) x is a subset of y} X
5) R: {(x,y), x is brother of y} X
6) R: $\{(x,y), (x \perp y)\}$ X
7) R: $\{(x,y) : (x+y=10)\}$ X.

Reflexive ⇒ Not Asymmetric → True

Symmetric ⇒ Not Asymmetric → True.

7) Equivalence relation:-

A relation which is reflexive, Symmetric and Transiti
is said to be an equivalence relation.

5 SUN 1) R: $\{(x,y) \mid x \parallel y\}$ ✓

2) R: $\{(x,y) \mid x \perp y\}$ X

3) R: $\{(x,y) \mid x \neq y\}$

4) R: {(a,b) iff a+b is even over the set of integer} ✓

5) R: {(a,b) iff a+b is odd over the set of integer} X

6) R: {(a,b) iff a.b>0 over the set of non zero rational ~~natural~~ number} ✓

7) R: {(a,b) iff $|a-b| \le 2$ over the set of natural number} X

4) :- a+a is even.
a+b is even → b+a is even.

6 MON a+b is even && b+c is even ⇒ a+c is even

even even	even even	even+even = even
odd odd	odd odd	odd+odd = even.

6) a.b > 0 && b.c > 0 ⇒ a.c > 0

-ve -ve	-ve -ve	-ve -ve
+ve +ve	+ve +ve	+ve +ve

7) $|a-b| \le 2$ && $|b-c| \le 2$ ⇒ $|a-c| \le 2$.

a = 10, b = 8, c = 7.

$|10-8| \le 2$ && $|8-7| \le 2$

$|10-7| \cdot 3 \le 2 \times \times$

Partial order relation :-

A relation which is reflexive, Antisymmetric and Transitive is P.O.R

$R_1 = \{(x,y) \mid x \le y\}$ ✓
$R_2 = \{(x,y) \mid x \subseteq y\}$ ✓
$R_3 = \{(x,y) \mid x/y\}$ ✓

Closure of a Relation :-

1) Reflexive closure
2) symmetric closure
3) Transitive closure.

1) Reflexive closure :-

find reflexive closure of R

$R = \{(1,1), (2,2), (2,3)\}$

Then S is said to be reflexive closure of R if
i) S is reflexive.
ii) $R \subseteq S$.
iii) S will be minimum such set.

$S = \{(1,1),(2,2),(2,3),(3,3)\}$ ✓

9 THU

$S' = \{(1,1),(2,2),(2,3),(3,3),(1,2)\}$ X.

2) Symmetric closure:-

S is said to be the symmetric closure of R if

i) S is symmetric

ii) $R \subseteq S$.

iii) S will be minimum such set.

$S = \{(1,1),(1,2),(3,3),(2,1)\}$ ✓

$S' = \{(1,1),(1,2),(3,3),(2,1),(2,2)\}$ X.

$S'' = \{(1,1),(1,2),(2,1)\}$ X.

10 FRI

3) Transitive closure:-

S is said to be the transitive closure of R

i) S is Transitive

ii) $R \subseteq S$

iii) S will be minimum such set.

Gate 1989 :-

$R = \{(1,2), (2,3), (3,4), (5,4)\}$
over $A = \{1,2,3,4,5\}$.

$S = \{(1,2), (2,3), (1,3), (3,4), (2,4), (5,4), (1,4)\}$

Congruence Modulo Relation :-

$a \equiv b \bmod c$
$8 \equiv 2 \bmod 3$
$(8-2) \% 3 = 0$

$\}$ → Is it a equivalence relation or not?

$8 \equiv -4 \bmod 3$
$(8-(-4)) \% 3 = 0$

$8 \equiv 4 \bmod 3$ X X.

$R = \{a, b\}$.

$R = \{(x,y) \mid y = x^i\}$ → is it a equivolence relation or not?

13 MON

Partition of a set

$A = \{1,2,3,4\}$.

$\pi_1 = \{1,2,3\}\ \{4\}$

$\pi_2 = \{1,2\}\ \{3,4\}$

$\pi_3 = \{1,4\}\ \{2,3\}$

i) $S_1 \cup S_2 \cup \ldots S_n = S$

ii) $S_1 \cap S_2 = \emptyset$ ~~&~~ $S_2 \cap S_3 = \emptyset \ldots$

$A = \{1,2\}$.

$\pi_1 = \{1\}\{2\}$

$\pi_2 = \{1\ 2\}$.

14 TUE

$A = \{1,2,3\}$

$\pi_1 = \{1\}\ \{2,3\}$

$\pi_2 = \{1,2\}\ \{3\}$

$\pi_3 = \{1,3\},\{2\}$

$\pi_4 = \{1,2,3\}$

$\pi_5 = \{1\}\{2\}\{3\}$

$$\frac{3!}{3!} + \frac{3!}{2!\times 1!} + \frac{3!}{1!\times 1!\times 1!\times 3!} = 5.$$

$A = \{1,2,3,4\}$

$$= \frac{4!}{4!} + \frac{4!}{3!\times 1!} + \frac{4!}{2!\times 2!\times 2!} + \frac{4!}{1!\times 1!\times 1!\times 1!\times 4!} + \frac{4!}{2!\times 1!\times 2!}$$

$= 15.$

Notes

$A = \{1,2,3,4,5\}$.

$$\frac{5!}{5!} + \frac{5!}{4! \times 1!} + \frac{5!}{3! \times 2!} + \frac{5!}{2! \times 2! \times 1! \times 2!} + \frac{5!}{3! \times 1! \times 1! \times 2!}$$

$$+ \frac{5!}{2! \times 1! \times 1! \times 1! \times 3!} + \frac{5!}{1! \times 1! \times 1! \times 1! \times 5!}$$

$= 1 + 5 + 10 + 15 + 10 + 10 + 1$

$= 52$

Quotientset :-

$A = \{1,2,3,4\}$

A/R :

$R = \{(1,1), (1,2), (2,1), (3,3), (4,4)\}$

R-Relative Set

$A = \{1,2,3\}$

$R = \{(1,1), (1,2), (2,3), (3,2)\}$

$R[1] : \{1,2\}$

$R[2] : \{3\}$

$R[3] : \{2\}$

R relative to A :- $R(A) = \{1,2,3\}$

17 FRI

$A = \{1,2,3\}$

$R = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$.

find A/R :-

$R[1] = \boxed{\{1,2\}}$

$R[2] = \boxed{\{1,2\}}$

$R[3] = \boxed{\{3\}}$

$$\begin{array}{ccc} & \{1,2,3,4\} & \\ \swarrow & & \searrow \\ \{1,2\} & & \{3\} \end{array}$$

$S = \{ball, bat, cat, call, catch\}$

18 SAT

$R = \{(w_1, w_2) \mid w_1 \text{ \& } w_2 \text{ starts with same symbol}\}$

$R = \{(ball, bat), (bat, ball), (cat, call), (call, cat),$
$(cat, catch), (catch, cat), (call, catch),$
$(catch, call), (ball, ball), (bat, bat), (cat, cat), (call, call),$
$(catch, catch)$

$R[catch] = \{catch, call, cat$

$R[ball] = \{bat, ball\}$

$R[bat] = \{ball, bat\}$

$R[cat] = \{cat, call, catch\}$

$R[call] = \{call, cat, catch\}$

Notes

$R = \{(w_1, w_2) \mid |w_1| = |w_2|\}$

$S/R = \{$ball, call$\}$ $\{$bat, cat$\}$ $\{$catch$\}$

Number of equivalence classes = 3.

Theorems:-

$R = \{(1,1), (2,2), (3,3), (1,2)\} \rightarrow$ Reflexive

$R^{-1} = \{(1,1), (2,2), (3,3), (2,1)\} \rightarrow$ Reflexive.

i) If R is Reflexive R^{-1} is also reflexive.
ii) If R is Symmetric R^{-1} is also Symmetric.
iii) If R is transitive then R^{-1} is also transitive.
iv) If R is equivalence then R^{-1} is also equivalence.
v) If R and S are two reflexive relation on set A then $R \cup S$ and $R \cap S$ both are reflexive.

vi) If R and S are symmetric relation on set A then $R \cup S$ and $R \cap S$ both are symmetric.
vii) If R and S are two transitive relation then $R \cap S$ is transitive however $R \cup S$ may / may not be Transitive.
viii) If R and S are two equivalence relation then $R \cap S$ will be a equivalence relation however $R \cup S$ may or may not be.

Notes

21 TUE

Enumeration on Relation

Suppose $|A| = m$, $|B| = n$

Then $|A \times B| = m \times n$.

Total number of relation $= 2^{mn}$

minimum number of elements in reflexive relation over set A with n element = n.

maximum number of element in reflexive relation with n element $= n^2$.

Total number of reflexive relation over set A with n element $= 2^{n^2 - n}$

22 WED

Total number of Symmetric relation $= 2^{\frac{n(n+1)}{2}}$

Total number of irreflexive relation. $2^{n^2 - n}$

Total number of Antisymmetric relation $= 2^n \cdot 3^{\frac{n^2 - n}{2}}$

Total number of Asymmetric relation $= 3^{\frac{n^2 - n}{2}}$

Total number of reflexive and symmetric relation $= 2^{\frac{n^2 - n}{2}}$

Notes

Lattice and Boolean Algebra.

1) Poset : A set along with partial order relation is called Poset.

P.O.R :- A relation which is reflexive, Antisymmetric, Transitive.

$(Z, \leq)$

Reflexive:- $\forall x \in Z \ xRx \Rightarrow x \leq x$.

Anti:- if $xRy \Rightarrow y \not R x$ unless $x = y$.

$x \leq y \Rightarrow y \nleq x$

Transitive:- $x \leq y$ and $y \leq z \Rightarrow x \leq z$.

$\therefore$ $(Z, \leq)$ is a Poset.

$(P(A), \subseteq)$

Reflexive:- $\forall x \in P(A) \ x \subseteq x$.

Antisymmetric:- $x \subseteq y \Rightarrow y \nsubseteq x$ unless $x = y$.

Transitive:- $x \subseteq y$ and $y \subseteq z \Rightarrow x \subseteq z$.

$(D_{20}, |)$

$(\{1, 2, 4, 5, 10, 20\}, |)$

D_{20} :- Divisors of 20.

25 SAT Reflexive :- 1/1, 2/2, 4/4 . . .

Anti Symmetric :- 1/2 ⇒ 2/1 false
4/20 ⇒ 20/4 false

Transitive :- 2/4 and 4/20 ⇒ 2/20

(D₈₁, I) → Poset .

Toset :- A Poset is said to be a toset if all element are Comparable .

({1, 3, 9, 27, 81}, I) → Toset 1/3 or 3/1 True
1/81 or 81/1 True .

26 SUN

({1, 2, 4, 5, 10, 20} I) → NOT Toset 4/5 or 5/4 False

P(P(A) ⊆) → Not a Toset

{∅, {1}, {2}, {1,2}}
{1} ⊄ {2} OR
{2} ⊄ {1} } False .

$(Z, \leq) \to$ Toset
$(Z^+, \leq) \to$ Toset.

Toset :- Total ordered set.

Woset :- well ordered set.
A Toset is said to be a woset if least element exists.

Every finite Toset is a woset $\to$ True.
Only finite Toset is a woset $\to$ False.

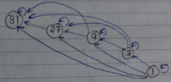
$(Z, \leq) \to$ woset ✗
$(Z^+, \leq) \to$ woset ✓
$(D_{81}, 1) \to$ woset ✓

i) Every Toset is a Poset. $\to$ True
ii) Every woset is a Toset $\to$ True
iii) Every poset is a Toset. $\to$ False
iv) Every Toset is a woset. $\to$ False.

HASSE DIAGRAM.

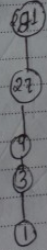
$(D_{81}, 1)$
$\{1, 3, 9, 27, 81\}$.

29 WED R = {(1,1) (1,3) (1,9) (1,27), (1,81)
(3,3) (3,9) (3,27) (3,81)
(9,9) (9,27) (9,81)
(27,27) (27,81), (81,81) }.



HASSE DIAGRAM :-

1) Remove self loop
2) Remove Arrow that shows transitive relation.
3) Remove Arrow Head.

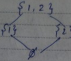


Poset

$\boxed{\text{Chain}}$

Notes

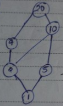
A = {1, 2}
(P(A) ⊆)



P(A) = {∅, {1}, {2}, {1, 2}}

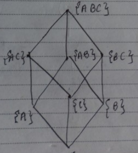
Toset X

(D_{20} |)
(1, 2, 4, 5, 10, 20)



Toset X   woset X

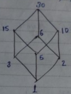
S = {A, B, C}
P(S), ⊆



3 SAT

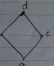
$(D_{30}, 1)$

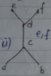
$\{(1,2,3,5,6,10,15,30), 1\}$



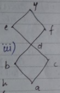
d, a

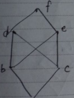
4 SUN

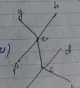
i)


ii)


e, f

iii)


iv)


v)


unique

i) Greatest ele.
ii) Least ele.
iii) upper bound
iv) Lower bound.
v) min element
vi) max element

Notes

i) Greatest, max = d
Least, min = a

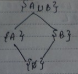
ii) Greatest, Least - doesnot exist
max - e, f
min = a, b

iii) Greatest, max = g, g
Least, min = a, d

iv) Greatest, max = f
Least, min = a

v) Greatest, Least → doesnot exist
max - {g, h, d}
min = {a, b, f}

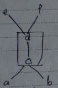
7 WED



UB{∅} = {∅, {A}, {B}, {A∪B}}

LB{A∪B} = {A∪B, A, B, ∅}.

LUB{A, B} = {A, B}
join (A, B)



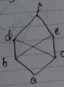
LUB(c, d) = d
GLB(c, d) = c.

8 THU

LUB → join
GLB → meet

LUB(a, b) = c
GLB(a, b) = doesnot exist.

LUB, GLB } Must be unique.



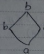
GLB(b, c) = {a}
LUB(b, c) = {d, e}
↳ doesnot exist.

Lattice :- A poset is said to be a lattice if GLB and LUB exist for all $(a, b) \in L$.

$(L, \wedge, \vee)$

$\wedge$ ↓ Meet (GLB)

$\vee$ ↓ Join (LUB)



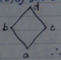
→ lattice ✓

ii) Not a Lattice
iii) Lattice
iv) Not a lattice
v) Not a lattice.

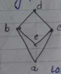
Lattice :- $(L, \wedge, \vee)$ → join or supermum.

(meet or infimum over $\wedge$)

A poset is said to be lattice if for every pair $a, b \in L$ GLB(a,b) and LUB(a,b) exists.



GLB(b,c) = a
LUB(b,c) = d.
∴ Lattice ✓.



GLB(a,e) = exist
LUB(a,e) = b/c

Lattice X

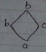
Notes

Lattice :- A poset is said to be a lattice if GLB and LUB exist for all $(a, b) \in L$

$(L, \wedge, \vee)$

$\wedge$ ↓ Meet (GLB)

$\vee$ ↓ Join (LUB)



→ lattice ✓

ii) Not a Lattice

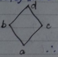
iii) Lattice

iv) Not a lattice

v) Not a lattice.

Lattice :- $(L, \wedge, \vee)$ — $\wedge$: meet or infimum; $\vee$: join or supremum.

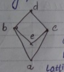
A poset is said to be lattice if for every pair $a, b \in L$ GLB(a,b) and LUB(a,b) exists.



GLB(b,c) = a

LUB(b,c) = d.

∴ Lattice ✓.



GLB(a,e) = e

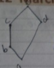
LUB(a,b) = b,c

Lattice ✗

Notes

11 SUN

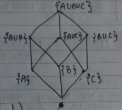
Lattice X



Lattice ✓

$GLB(b,d) = a$

$LUB(b,d) = e$

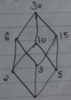


$LUB(A,C) = A \cup C$

$GLB(A,C) = \phi$

$(D_{30}, |)$.

12 MON



$GLB(6,5) = 1$ → $HCF(6,5)$

$LUB(6,5) = 30$ → $LCM(6,5)$

$HCF(3,10) = 1$

$LCM(3,10) = 30$.

({1, 3, 5, 10, 15, 30} /)

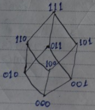
→ It is a Po-Set.

GLB(3, 10) = 1
LUB(3, 10) = 30

LUB(3, 5) = 15
GLB(3, 5) = 1

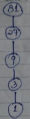
Ques :-

Draw Hasse Diagram for 3-bit Binary number whose adjacent distance is 1 and find whether it is a Lattice or not.



Ques :- {1, 3, 9, 27, 81 | }

15 THU whether it is a lattice or not.



Poset
Toset
Woset
Lattice

1) Every poset is a lattice. False
2) Every Toset is a lattice True
3) Every Woset is a lattice True.
4) Every Lattice is a Toset False.

16 FRI

Properties followed by Lattice :-

1) Idempotent law :-

$(a \vee a)$ a join a = a

$(a \wedge a)$ a meet a = a.

2) Commutative law :-

$a \vee b = b \vee a$

$a \wedge b = b \wedge a$.

3) Associative law :-

$a \vee (b \vee c) = (a \vee b) \vee c$.

$a \wedge (b \wedge c) = (a \wedge b) \wedge c$.

iv) Closure property :-

for any element $a, b \in L$.

$a \wedge b \in L$

$a \vee b \in L$

Properties need not to be followed by a lattice :-

1) Complement
2) Distributive
3) Identity.

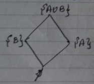
And Complement in a lattice :-

Bounded Lattice :- $(L, \vee, \wedge, 0, 1)$ Lowest element highest element.

A lattice is said to be a bounded lattice if 0 and 1 exists for it for any element $a \in L$.

$a \vee 0 = a$

$a \wedge 1 = a$.



$\{A\}$ join $\emptyset = \{A\}$

$\{A\}$ meet $\{A \cup B\} = \{A\}$.

19 MON

→ Every finite lattice is a bounded lattice.

$(Z, \leq) \rightarrow$ not a bounded lattice

$(Z^+, \leq) \rightarrow$ not a bounded lattice.

$S = \{A, B, C\}$

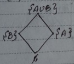
1) $(P(S), \subseteq) \rightarrow$ bounded lattice.
2) $(D_n, |)$ where n is finite number → bounded lattice
3) $(D_n, |)$ where n is prime number → bounded lattice

for any $a \in L$ $\exists a'$ such that :-

I) a join $a' = 1$ (highest element).
II) a meet $a' = 0$ (lowest element).

$(a)^c = a'$.

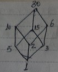
20 TUE



$\{A\}^c = B$
$\{B\}^c = A$.
$(\{A \cup B\})^c = \emptyset$
$(\emptyset)^c = \{A \cup B\}$.

→ If any lattice is containing of complement of all elements then that lattice is known as complementary lattice.

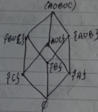
$(D_{30}, 1)$



$(2)^c$

$LCM(2, \overset{15}{x}) = 30$

$HCF(2, \underset{15}{x}) = 1$

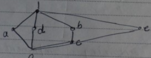


meet

$(B \cdot \{A \cup C\}) = \emptyset$.

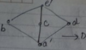
$(B \text{ join } \{A \cup C\}) = \{A \cup B \cup C\}$

1988



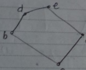
Complement of a = { b, c, d, e. }

23 FRI



$\rightarrow$ Diamond

$(c)^c = \{b, d\}$



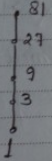
$\rightarrow$ Pentagon

$(c)^c = \{b, d\}$

$(a)^c = b$

$(b)^c = a$.

24 SAT



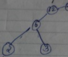
$(9)^c = X$ does not exists.

1) Every Toset is a Complementary lattice → False
2) Every finite Toset is a Complementary lattice → False
3) Every Toset of two element is a complementary lattice → Tr
4) Every bounded lattice is Complementary → False.

Ex:- Let $X = \{2, 3, 6, 12, 24\}$

Let $\leq$ be P.O.R defined by $x \leq y$.

if x divides y. Then numbers of edge in hasse diagram.



a) 8 ~~b) 4~~ c) 5 d) 9.

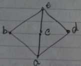
Distributive Lattice :-

A lattice is said to be distributive if for $a, b, c \in L$

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$
$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c).$$

$\vee \to$ join , $\wedge \to$ meet.

1)



$b \wedge (c \vee d) = (b \wedge c) \vee (b \wedge d)$

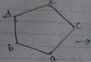
$b \wedge (c \vee d) = b \wedge e$
$= b.$

$(b \wedge c) \vee (b \wedge d) = a \vee a$
$= a$

$\therefore a \neq b$ $\therefore$ It is not a distributive lattice

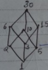
Notes

27 TUE



→ It is also not a distributive lattice.

Note :- If any lattice contain sublattice equivalent to diagram I or diagram II then that lattice is not a distributive lattice.



→ It is a distributive lattice.

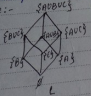
28 WED Sublattice $(L', \wedge, \vee)$

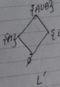
L' is said to be a sublattice of L if :-

i) $L' \subseteq L$

ii) L' has GLB and LUB for each pair $a, b \in L'$.

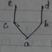
for example :-

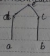




Semilattice

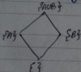
MeetSemilattice JoinSemilattice.



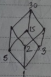


Boolean Algebra :-

A Complementary, distributive lattice is a Boolean Algebra.



BooleanAlgebra ✓



Booleanalgebra ✓

$(D_{23}, 1)$.

Booleanalgebra ✓

$(D_{20}, 1)$ $20 \to 2 \times 2 \times 5$

$\{ \{1, 2, 4, 5, 10, 20\}, 1 \}$

$LCM(10, \overset{2}{x}) = 20$

$HCF(10, 2) = 2. X$

$(10)^c = X.$

$\therefore$ It is not a Boolean algebra.

$(D_n, |) \rightarrow$ It is a Boolean algebra if

1) n is a prime number.
2) prime factors of n are non-repetitive.

Group Theory.

$(S, *)$: A set along with binary operation is called algebraic structure

Binary operation:-
For any two element x, y $x * y$ exists, unique.

$S = Z$.

$*(x,y) = x+y$ ✓
$*(x,y) = x \times y$ ✓
$*(x,y) = \min(x,y)$ ✓
$*(x,y) = \max(x,y)$ ✓
$*(x,y) = x$ ✓
$*(x,y) = \sqrt{xy}$. XX.

1) ~~Monoid~~ Semigroup :- An algebraic structure is said to be ~~monoid~~ semigroup if

i) It is closed.

ii) It is associative.

Closed :- $\forall a,b \in Z \quad a*b \in Z$

$(Z, +) \rightarrow$ closed

$(R, +) \rightarrow$ closed.

$(Z, -) \rightarrow$ closed

$(Z^+, -) \rightarrow$ Not closed. $\rightarrow$ e.g. $= -15+10 = -5 \notin Z^+$

Associative :- $\forall a,b,c \in S$

$(a*b)*c = a*(b*c)$.

$(Z,+)$

$(2+3)+4 = 2+(3+4)$

$5+4 = 2+7$

$9 = 9$.

$(Z,+) \rightarrow$ Semigroup ✓

$(Z,-) \rightarrow$ Semigroup ✗

1) Monoid :- An algebraic structure is said to be Monoid if

i) It is closed

ii) It is associative.

iii) It has a identity element.

identity element :-

$\forall a \in S$.

$a*e = e*a = a$.

and e must be unique for entire set also belongs to S.

i) $(Z,+)$ ii) $(R,+)$ iii) $(R,\times)$

3 TUE iv) $(Z,*)$ such that $*(x,y) = x-y$

(ii) $(R,+)$:- Closed ✓ (i) monoid ✓

associative ✓

identity element ✓

∴ monoid.

(iii) monoid ✓ (iv) closed ✓, associative ✗

monoid ✗

(v) $(Z,*)$ such that $\min(x,y) = *(x,y)$.

$$x*e = \min(x,e) = x$$
$$e*x = \min(e,x) = x \Rightarrow e = +\infty.$$

4 WED

$+\infty \neq Z$.

(vi) $(Z^+,*)$ " " $*(x,y) = \min(x,y)$ monoid ✗.

$e = +\infty$.

(vii) $(Z^+,*)$ " " $*(x,y) = \max(x,y)$. monoid ✓

$e = 1$.

(viii) $(Z,*)$ " " $*(x,y) = \max(x,y)$ monoid ✗

$e = -\infty$.

(ix) $(R,*)$ " " " monoid ✗

$e \to$ does not exist.

Notes

$(R\ *)$ $\quad *(x,y) = x+y-xy$.

$x+e-xe = x$

$e(1-x) = 0 \quad \Rightarrow e = \frac{0}{(1-x)}$

$\Rightarrow e = 0.$

$e+x-ex = x$

$e(1-x) = 0$

$\Rightarrow e = \frac{0}{(1-x)}$

but at $x = 1$ e is not defined.

$\therefore$ It is not a monoid.

But if given that $(R-\{1\}, *)$.

$(R\ *)$ — semigroup?

$*(x,y) = x+y-xy$.

For any two real number x and y

Closed

$\underbrace{x+y}_{R} - \underbrace{xy}_{R} \in R$.

$R - R = R \in R$

Associative :-

$(x*y)*z = x*(y*z)$

L.H.S :-

$(x+y-xy)*z$

$= x+y-xy+z-(x+y-xy)z$

$= x+y-xy+z-xz-yz+xyz$

R.H.S :-

$x*(y+z-yz)$

$= x+y+z-yz-x(y+z-yz)$

$= x+y+z-yz-xy-xz+xyz$

∴ L.H.S = R.H.S . ∴ It is a Semigroup.

$(R, *)$

$*(x,y) = xy+x-y$

closed ✓

Associative :-

8 SUN $(x*y)*z$

$= (xy+x-y)*z$

$= (xy+x-y)z + xy+x-y-z$

$= xyz+xz-yz+xy+x-y-z$

R.H.S :- $x*(y*z)$

$= x*(yz+y-z)$

$= x(yz+y-z)+x-yz-y+z$

$= xyz+xy-xz+x-yz-y+z$

∴ L.H.S ≠ R.H.S . ∴ Not Associative.

∴ It is not a Semigroup.

Ques: A binary operation ⊕ is defined over integers such that

$x \oplus y = x^2 + y^2$.

Then which of the following is true?

a) Commutative but not associative.
b) Associative but not Commutative.
c) Both Associative and Commutative.
d) Neither associative nor Commutative.

$(x \oplus y) \oplus z = (x^2+y^2) \oplus z = (x^2+y^2)^2 + z^2$
$= x^4 + y^4 + 2x^2y^2 + z^2$

$x \oplus (y \oplus z) = x \oplus (y^2+z^2)$
$= x^2 + (y^2+z^2)^2$
$= x^2 + y^4 + z^4 + 2y^2z^2$.

∴ NOT Associative

$a \oplus b = a^2 + b^2$
$b \oplus a = b^2 + a^2$.
∴ $a \oplus b = b \oplus a$.
∴ Commutative.

Monoid :- Algebraic Structure must be
1) Closed
2) Associative
3) Identity element
↳ It must be unique for S (entire Set)
↳ also $e \in S$.

11 WED

$(Z,+)$

for any element

$x * e * x = x$.

$$\left.\begin{array}{l} x+e = x \\ e+x = x \end{array}\right] e = 0 \in Z.$$

Consider the set of Σ^* of all string over the alphabet set $\Sigma = (0,1)$ along with Concatenation operation.

(a) Doesnot form a group
b) form a non-commutative group.
c) Doesnot have a right identity element.
d) form a group if empty string is removed from Σ^*

12 THU

closed:-

$\forall x,y \in \Sigma^*$

$x \cdot y \in \Sigma^*$ $\therefore$ Closed ✓.

Associative:-

$(x \cdot y) \cdot z = x \cdot (y \cdot z)$

$\therefore$ Associative ✓.

identity element $= \lambda$.

$\forall x \in \Sigma^*$

$x \cdot \lambda = \lambda \cdot x = x$.

$\therefore$ identity element ✓.

Notes

Group :- $(S, *)$

1) closed
2) associative
3) identity element
4) Inverse exists.

$a * a^{-1} = e$

$a * x = e$

$\Rightarrow (a)^{-1} = x$

Constraints :-
1) $a^{-1} \in S$
2) for every element a, a^{-1} must be unique

$a \to a^{-1}$, $a \to b^{-1}$ ✗ (NOT POSSIBLE)

$a \to a^{-1}$

$b \to b^{-1}$ ✓ (POSSIBLE)

$c \to c^{-1}$

$\omega \cdot \omega^{-1} = \lambda$

$\omega^{-1} = ?$

$\therefore$ ω^{-1} is not possible.

$\therefore$ It is not a Group.

1) $(Z, +)$:-

Inverse ?

$x + x^{-1} = 0$

$\Rightarrow x^{-1} = -x$

eg :- $5 \to -5 \in Z$

$10 \to -10 \in Z$

2) $(Z, \times)$:-

Inverse ?

$x \times x^{-1} = 1$

$\Rightarrow x^{-1} = \frac{1}{x} \notin Z$ $\therefore$ Inverse does not exists.

$\therefore$ It is not a group.

15 SUN

3) $(R, +)$:-

$x + x^{-1} = 0$.

$x^{-1} = -x \in R$. $\therefore$ Inverse exists

$\therefore$ It is a Group.

4) $(R, \times)$ $e = 1$

$y \times y^{-1} = 1$

$\Rightarrow y^{-1} = \frac{1}{y}$.

$\therefore$ Inverse exists for all elements except for 0.

$\therefore$ $(R, \times)$ is not a Group.

5) $(R - \{0\}, \times)$

$\longrightarrow$ It is a Group.

16 MON

Ques :- $(R, *)$

$*(x, y) = x + y - xy$.

$e = \frac{0}{1-x}$ if $x = 1$ {identity element does not exists}.

$\therefore$ It is not a Group.

Ques :- $(R, *)$ $*(x, y) = x$.

Closed ✓

$(x * y) * z = x * z = x$

$x * (y * z) = x * y = x$

$\therefore$ Associative ✓

Notes

$* \quad o(x^{-1} a x) = o(a)$.

$f(x,e) = *(e,x) = x$

$x * e = x$ — (i)
$e * x = e$ — (ii) } can't find e

∴ Identity element ✗

Abelian Group :-

A group is said to be Abelian if it is Commutative.

i) closed ii) Associative iii) identity element exists
iv) Inverse exists v) Commutative.

$(Z, +) \to$ abelian Group
$(Z, X) \to$ monoid
$(R, +) \to$ abelian Group
$(R, X) \to$ monoid
$(R - \{0\}, X) \to$ abelian Group

$(R *)$ $*(x,y) = \min(x,y) \to$ Semigroup
$(R *)$ $*(x,y) = \max(x,y) \to$ Semigroup.
$(Z - \{0\}, X) \to$ monoid.
$(\Sigma^*, \cdot) \to$ monoid.
$(Z^+, *)$ $*(x,y) = \max(x,y) \to$ monoid.
$(R, *)$ $*(x,y) = x + y - xy \to$ Semigroup.

CALEY TABLE :-

Representation of algebraic structure into a table is called C.T.

Notes

*	a	b	c
a	a	b	c
b	b	c	a
c	c	b	a

1) Closed ✓

2) $(a * b) * c = a * (b * c)$.

$b * c = a * a$.

$a = a$.

$\therefore$ Associative ✓.

3) identity element :-

$e = a$.

4) Inverse :-

$(a)^{-1} = a$ $\quad [\because e^{-1} = e]$.

$(b^{-1}) = c$

$(c^{-1}) = c$.

5) Commutative :-

$b * c = a$

$c * b = b$ $\quad \therefore \; b * c \neq c * b$.

$\therefore$ It is a Group but not an abelian Group.

Ques: - Cube root of unity $(1, \omega, \omega^2)$

$\times$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

1) Closed ✓
2) Associative ✓
3) identity element , $e = 1$ ✓
4) Inverse :- ✓

$1 - 1$
$\omega - \omega^2$
$\omega^2 - \omega$.

5) Commutative ✓

$\therefore$ Abelian Group ✓

Ques:-

$\times$	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	$+1$
$-i$	$-i$	$+i$	1	-1

1) Closed ✓
2) Associative ✓
3) identity element , $e = 1$ ✓
4) Inverse

$1 \to 1$
$-1 \to -1$
$i \to -i$
$-i \to i$

5) Commutative ✓

$\therefore$ Abelian Group ✓

23 MON

Ques :- $\{1, 2, 3, 5, 7, 8, 9\}$ under multiplication ~~modulo~~ modulo 10 is not a Group. Given below are four ~~four~~ possible reasons which of the following is false?

1) It is not closed.
2) 2 does not have an inverse
~~3)~~ 3) 3 does not have an inverse.
4) 8 does not have an inverse.

X_{10}	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	0	2	4	6	8
3	3	6	9	2	5	8	1	4	7
4	4	8	2	6	0	4	8	2	6
5	5	0	5	0	5	0	5	0	5
6	6	2	8	4	0	6	2	8	4
7	7	4	1	8	5	2	9	6	3
8	8	6	4	2	0	8	6	4	2
9	9	8	7	6	5	4	3	2	1

24 TUE

Sol - $\{1, 2, 4, 7, 8, 11, 13, 14\}$ → multiplication modulo 15, inverse of 4 and 7?

$(4)^{-1} = 4$

$(7)^{-1} = 13$

Notes

Note :- If $a^{-1} = b$ then $b^{-1} = a$.

Ques :- $\{\{1, 2, 3, 4, 5, \ldots p-1\} \times \text{modulo } P\}$

where P is a prime no.

p = 5.

X	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

1) Closed ✓
2) associative ✓
3) Identity element ✓
4) inverse ✓
5) Commutative

∴ It is a Abelian Group.

Note :- It is always an abelian group for any prime number p.

Ques :- which of the following is not necessary the property of a group.

1) Associativity
2) existence of inverse for every element
3) existence of identity.
4) Commutative

Ques :- Consider the set S of all 3×3 matrices

27 FRI

$$\begin{bmatrix} a & f & e \\ 0 & b & d \\ 0 & 0 & c \end{bmatrix}$$ given $a, b, c, d, e, f \in R$ & $abc \neq 0$.

is under the multiplication operation the set is

(a) group b) monoid but not Group c) Semi group but not monoid d) neither group nor monoid.

Ques:-

*	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b			
c	c	e		

28 SAT

The above table is incomplete operational table of a 4 element group. The last Row of the table is.

a) c a e b
b) c b a e
c) c b e a
(d) c e a b

Cyclic Group :-

A group $(G, *)$ is said to be cyclic $\forall a \in G\ \exists g \in G$ such that $g^n = a$.

29 SUN

$\times$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

$(\omega)^0 = 1$

$(\omega)^1 = \omega$

$(\omega^2) = \omega \times \omega = \omega^2$

$\therefore \omega$ is a generator.

$(\omega^2)^0 = 1$, $(\omega^2)^1 = \omega^2$, $(\omega^2)^2 = \omega^2 \times \omega^2 = \omega \cdot \omega^3 = \omega$.

$\therefore g$ is a generator. $\therefore g = \{\omega, \omega^2\}$

30 MON

→ Atleast one generator implies that group is cyclic.

Note :- * Identity element can never be a generator because any power of identity element is the identity element itself.

* If a is a generator then a^{-1} is also a generator.

generators for a group always exists in pair :- false. because a element and its inverse can be same.

Notes

1 TUE

*	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

$(b)^0 = a$, $(b)^1 = b$, $(b)^2 = b * b = c$

$\therefore$ b is a generator

$b * b^{-1} = e = a$

$\Rightarrow b^{-1} = c$

$\therefore$ c is also a generator.

$g = \{b, c\}$.

$\therefore$ The above group is cyclic.

2 WED

$+_4$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

$0 = 4$.

$(1)^1 = 1$ $(1)^2 = 2$ $(1)^3 = 3$ $(1)^4 = 0$.

$(1)^{-1} = 3$

$(2)^1 = 2$, $(2)^2 = 0$ $(2)^3 = 2$ $(2)^4 = 0$

Something to the power 0 is identity element.

Notes

Gate2009:-

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

$0 = 4.$

$e = a.$

$(c)^0 = a \ , \ (c)^1 = c \ , (c)^2 = b \ , \ (c)^3 = d.$

$\therefore$ c is a generator

$(c)^{-1} = d.$

$\therefore$ d is also a generator.

$g = \{c, d\}.$

X	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

$e = 1$

$(-1)^1 = -1 \ , \ (-1)^2 = 1 \ , \ (-1)^3 = -1$

$(i)^0 = 1 \ , \ (i)^1 = i \quad (i)^2 = -1 \quad (i)^3 = -i$

$(i)^{-1} = -i.$

$g = \{ i, -i \}.$

5 SAT

Que :-

X_p	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

$O = 4$

$(2)^0 = 1$, $(2)^1 = 2$, $(2)^2 = 4$, $(2)^3 = 3$.

$(2)^{-1} = 3$.

$\therefore g = \{2, 3\}$

Note :- Every cyclic Group is abelian however Converse may not be true.

Que:- $(Z, +)$, is it cyclic Group ?

6 SUN

Power of an element of a group :-

i) $(a)^0 = e$ ii) $(a)^1 = a$ iii) $a^2 = a * a$.

iv) $(a)^n = a * a * \ldots$ n times (v) $(a)^{-1} = a^{-1}$

(vi) $(a)^{-2} = (a^{-1})^2 = a^{-1} * a^{-1}$ (vii) $a^{-3} = (a^{-1})^3$

$= a^{-1} * a^{-1} * a^{-1}$

$(a)^{m+n} = a^m * a^n = (a * a * a \ldots m \text{ times})$

$* (a * a * a \ldots n \text{ times}$

$(1)^0 = 0$

$(1)^1 = 1$

$(1)^2 = 1+1 = 2$

$(1)^3 = 1+1+1 = 3$

$(1)^{-1} = -1$

$(1)^{-2} = 1^{-1} + 1^{-1} = -1 - 1 = \ldots$

$(1)^{-3} = 1^{-1} + 1^{-1} + 1^{-1}$

$= -1 - 1 - 1 = -3$.

Notes

Sun	Mon	Tue	Wed	Thu	Fri
1	2	3	4	5	6
8	9	10	11	12	13
15	16	17	18	19	20
22	23	24	25	26	27
29	30				

(ii) $(R, +)$
→ NOT cyclic
however it is abelian.

(iii) $(R - \{0\}, \times)$
→ Not cyclic.

ORDER OF a Group :-
$O(G)$:- No. of elements present in the group.

$G = \{\{1\ \omega\ \omega^2\}\ \times\}$ $\quad O(G) = 3$

$G_1 = (Z\ +)$ $\quad O(G_1) = \infty$
$G_2 = (R\ +)$ $\quad O(G_2) = \infty$.

Order of an element of the group.

$(G\ \times)$
$O(a) = n$ if $a^n = e$.
n must be a +ve integer.
(n cannot be zero.)

$\omega^1 = \omega$
$\omega^2 = \omega^2$.
$\omega^3 = \omega^3 = 1$ $\quad \therefore \quad O(\omega) = 3$.

$(\omega^2)^1 = \omega^2$ $\quad (\omega^2)^3 = 1$
$(\omega^2)^2 = \omega$ $\quad O(\omega^2) = 3$

9 WED

$(1)^1 = 1$.

$o(1) = 1$.

→ order of identity element is 1.

$\times$	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

$o(1) = 1$ $\quad o(-1) = 2$ $\quad o(i) = 4$

$o(-i) = 4$.

$G = (Z, +)$, $o(1) = ?$

10 THU

$o(1) = \infty$ $\quad o(0) = 1$.

$o(2) = \infty$

⋮

1) $o(a) \le o(G)$ - True

2) $o(a) \div o(G)$ - True.

3) $o(a) = o(a^{-1})$ - True.

4) $o(a) = x \Rightarrow a^m = e$ if $m = kx$ - True.

5) $o(a*b) = o(b*a)$ - True

or. $o(ab) = o(ba)$

Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

$\oplus$	0	1
0	0	1
1	1	0

closed ✓ , associativity ✓ , identity element = 0

$(0)^{-1} = 0$. inverse exists ✓ , commutative ✓

$(1)^{-1} = 1$.

$g = \{1\}$ $\quad 1^0 = 0 \;,\; 1^1 = 1 \;,\; 1^2 = 0 , \ldots$

$\therefore$ The Group is cyclic.

Note :- If $\forall a \; (a)^{-1} = a$ then that group is a abelian group.

Ques :- $(G, \circ)$ is a Group and is known to be abelian. then which of the following is true?

a) $g = g^{-1}$ for every $g \in G$

b) $g = g^2$ for every $g \in G$

c) $(g \circ h)^2 = g^2 \circ h^2$ for every $g, h \in G$

d) G is of finite order.

$(a * b)^2 = (a * b) * (a * b)$

$= (a * a) * (b * b)$

$= a^2 * b^2$.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

June 2012

13 SUN

Properties of a Group :-

i) closed ✓ ii) Associative

iii) identity element iv) for every element inverse must be unique v) $(a^{-1})^{-1} = a$

vi) $(ab)^{-1} = b^{-1} a^{-1} \quad (b^{-1} * a^{-1})$

vii) $ax = b \Rightarrow x = a^{-1} * b$

$ya = b \Rightarrow y = b * a^{-1}$

viii) $a * a^{-1} = e$.

Subgroup : $(H, *)$

$(H,*)$ is said to be a Subgroup of $(G,*)$ if

$$\left\{ (G,*) \begin{array}{l} \xrightarrow{\text{Subgroup}} \{e, *\} \\ \xrightarrow{\text{Subgroup}} (G,*) \end{array} \right\}$$

1) H is a subset of G.

2) H is a group itself.

14 MON

$+_4$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

$(\{0, +_4\})$ $\quad (\{0,2\} +_4) \quad (\{\{0,1,2,3\} +_4\})$.

$O(H) \div O(G)$ [It is always true for a Subgroup].

$(Z, +) \to$ Group
$(E, +) \to$ Subgroup ✓
$(O, +) \to$ Subgroup ✗
$(2Z, +) \to$ Subgroup ✓
$(3Z, +) \to$ Subgroup ✓
$(Z^+, +) \to$ Subgroup ✗

Normal Subgroup (NSG):-

$(H, *)$ is said to be Normal if:-
1) $(H, *)$ is a Subgroup of $(G, *)$
2) $aH = Ha$ $(\forall a \in G)$

left coset determined by a → Right Co-Set determined by a.

G:-

$+_4$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

H:-

$+_4$	0	2
0	0	2
2	2	0

$G = \{0, 1, 2, 3\}$

$0H = H0$
$1H = H1$
$2H = H2$
$3H = H3$.

$0H = 0\{0, 2\}$
$= 0 +_4 0 = 0$
$= 0 +_4 2 = 2$
$= \{0, 2\}$

$\{H0\} = \{0, 2\} 0$

17 THU

$0 +_4 0 = 0$

$2 +_4 0 = 2$

$1H = 1\{0,2\}$ $\{0,2\}$ $\therefore\ 0H = H0$

$1 +_4 0 = 1$

$1 +_4 2 = 3$

$H1 = \{0,2\}1$

$= 0 +_4 1 = 1$

$= 2 +_4 1 = 2+1 = 3$ $\therefore\ 1H = H1$

$2H = 2\{0,2\}$ $H2 = \{0,2\}2$

$= 2 +_4 0 = 2$ $= 0 +_4 2 = 2$

$= 2 +_4 2 = 0$ $= 2 +_4 2 = 0$

$\therefore\ 2H = H2$

Similary, $3H = H3$.

18 FRI <u>Note</u>:- Every Subgroup of an Abelian group is Norm

i) Homomorphism of Groups:-

Two Groups $(G_1, *_1)$ & $(G_2, *_2)$ defined by a function F is said to be Homomorphic if:-

$f(a *_1 b) = f(a) *_2 f(b)$.

$(R^+ \times) \xrightarrow{f(x) = \log x} (R\ +)$

G_1 G_2

$f(a) = \log a$, $f(b) = \log b$.

$f(a \times b) = \log(a \times b)$

Notes

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1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

$= \log a + \log b$

$= f(a) + f(b)$

Eg :-

$G_1 (R^+, x)$] $f(x) = e^x$

$G_2 (R, +)$

$f(x) = e^x$

$f(a) = e^a$

$f(b) = e^b$

$f(a \times b) = e^{a \cdot b} = e^{ab}$ ✗

∴ It is not a Homomorphic

ii) Monomorphism of Groups.

Two Groups $(G_1, *_1)$ & $(G_2, *_2)$ defined by a function f is said to be Monomorphic if

i) $f(a *_1 b) = f(a) *_2 f(b)$

ii) and $f(x)$ must be one-one.

i.e. Homomorphic + one-one.

$f(x) = \log x$ one-one ?

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$\log x_1 = \log x_2 \Rightarrow x_1 = x_2$

iii) Epimorphic of Groups:-

Two Group $(G_1, *_1)$ and $(G_2, *_2)$ defined by a function f is said to be epimorphic if

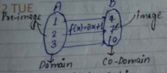
i) $f(a *_1 b) = f(a) *_1 f(b)$.

ii) and $f(x)$ must be onto.

iv) Isomorphic = Homomorphic + bijective function (one-one and onto function)

Function

A function is a rule that maps two set or, mapping of two set is a function or, a rule that associate or relate two set is a function.

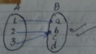


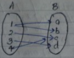
$f(x) = 3x+1$

$f(1) = 3\times1+1 = 4$ Range $\subseteq$ Co-Domain

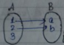
$f(2) = 7$

$f(3) = 10$





XX → Not a valid function.



XX
↳ Not a function.

$f(x) = \sqrt{x}$ → Not a valid function

$f(x) = +\sqrt{x}$ → a valid function

Rules :- 1) There can be no variable whose image is not present.

2) There cannot be two images or more for a single preimage.

$R_1 = \{(1,2), (2,b), (2,c), (3,a)\}$. XX

$R_2 = \{(1,a), (2,a), (3,a)\}$ ✓✓

$R_3 = \{(1,a), (2,b), (3,c)\}$ ✓✓

Domain and Range of a function:-

$f(x) = 3x+1$ is defined over $R \times R$

$f: R \times R$
A B

Domain for $f(x) = 3x+1$ is $\{R\}$

Domain for $f(x) = \log x$ is $\{R^+\}$.

Domain for $f(x) = e^x$ is $\{R\}$.

Notes Domain and Range are same for identity function.

25 FRI

Range :- $f(x) = 3x+1$.

$y = 3x+1$

$x = \frac{y-3}{1}$

$\therefore$ Range :- {R}.

$y = \log x$.

$x = e^y$ Range :- {R}.

$f(x) = 3x+1$ $\underset{D}{Z} \times \underset{\text{Co-Domain}}{Z}$

$y = 3x+1$.

$x = \frac{y-1}{3}$ R: $3K+1$ where k is a integer.

26 SAT

$R \subseteq$ Co-Domain.

Surjective function.

ONTO function :-

A function is said to be onto if $R =$ Co-Domain.

INTO function :-

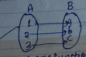
A function is said to be into if $R \subset$ Co-Domain.

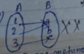
injective function.

ONE-ONE function :-

A function is said to be one-one if $f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$.

1) 

2) 

Notes

$\rightarrow$ One-one function. $\rightarrow$ NOT a one-one function.

$f(x) = 3x+1$.

$f(x_1) = 3x_1 + 1$

$f(x_2) = 3x_2 + 1$

$3x_1 + 1 = 3x_2 + 1 \Rightarrow 3x_1 = 3x_2 \Rightarrow x_1 = x_2$.

$f(x) = x^2$

$f(x) = x_1^2$, $f(x_2) = x_2^2$

$x_1^2 = x_2^2$

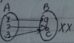
$\Rightarrow x_1 = \pm x_2$ $\therefore$ NOT a function (one-one)

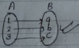
$f(x) = e^x$

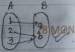
$f(x_1) = e^{x_1}$ $f(x_2) = e^{x_2}$

$e^{x_1} = e^{x_2}$ $\Rightarrow x_1 = x_2$.

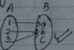
Onto function :-

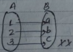


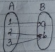




Many-one function :-





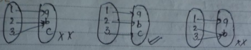


$\rightarrow$ If a function is not one-one $\therefore$ definitely a function is many-one.

Notes

29 TUE

Bijective function :- one-one + onto.



Which of the following are a bijective function ?

i) $f(x) = 3x+1$ $R \times R$ ✓

ii) $f(x) = 3x+1$ $Z \times Z$. $R = 3x+1 \subseteq Z$ XX

iii) $f(x) = \log x$ $R^+ \times R$ ✓

iv) $f(x) = e^x$ $R \times R$. X

v) $f(x) = e^x$ $R \times R^+$. ✓

Invertible function :-

30 WED

A function is invertible if it is bijective (one-one and onto).

$f(x) = 3x+1$

$y = 3x+1$

$x = \frac{y-1}{3}$

$f^{-1}(y) = \frac{y-1}{3}$

$\therefore f^{-1}(x) = \frac{x-1}{3}$.

Notes

Composition of function:-

$f(x) = 3x+1$

$g(x) = \sin x$

$fog(x) : f(g(x))$

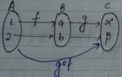
$f(g(x)) = 3(\sin x)+1$.

$gof(x) : g(f(x))$

$= \sin(3x+1)$

$f(g(x)) \neq g(f(x))$ $\therefore$ NOT COMMUTATIVE.

* $fof^{-1}(x) : x$
* $f^{-1}of(x) : x$



$f(1) = a$

$f(2) = b$

$g(a) = \alpha$

$g(b) = \beta$

$h = gof$

$h(1) = \alpha$

$h(2) = \beta$.

i) If f is one-one and g is one one then gof is one-one.

ii) If f is onto and g is onto then gof is onto.

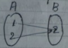
iii) If f is one-one and onto and g is one-one and onto then gof is one-one and onto.

iv) If gof is onto then g must be onto however

1 FRI

f need not to be onto.

Total number of function n^m or $|B|^{|A|}$



ii) one-one function : nP_m

iii) Onto function : $\boxed{n^m - n}$

iv) one-one and onto: $n!$ or $m!$

v) into function: $(n^m - (n^m - n!))^*$
set

Ques:- Let A be a finite size of n. Then the number of element in power set of $A \times A$.

$= 2^{n^2}$

2 SAT

Ques:- Let S be a infinite set and $S_1, S_2, S_3, \ldots S_n$ be set such that $S_1 \cup S_2 \cup \ldots \cup S_n = S$.

Then which of the following is true?

a) Atleast one of the set S_i is a finite set.

b) not more than one of the set S_i can be finite.

c) Atleast one of the set S_i is infinite.

d) not more than one set S_i is an infinite set.

Notes

Ques:- Let R be a symmetric and transitive relation on a set A then A) R is reflexive and hence an equivalence relation.
B) R is reflexive and hence partial order relation.
C) R is not reflexive and hence not an equivalence relation
(d) N.O.T.

Ques:- Suppose x and y are sets and |x| and |y| are respective cardinalities. It is given that there are exactly 97 function from x to y. then
(a) |x| = 1, |y| = 97
b) |x| = 97, |y| = 1.
c) |x| = 97, |y| = 97
d)

Ques:- Which of the following statement is false :-

(a) The set of Rational number is an abelian group under multiplication.
b) The set of integer is an abelian group under addition.
c) The set of Rational number form a abelian group under addition.
d) The set of Real no excluding zero is an abelian group under multiplication.

5 TUE

Ques:- Number of equivalence relation on a set $S = \{1,2,3,4\}$ is = number of partitions

$$= \frac{4!}{4!} + \frac{4!}{3! \times 1!} + \frac{4!}{2! \times 2! \times 2!} + \frac{4!}{1! \times 2! \times 1! + 2!} + \frac{4!}{1! \times 1! \times 1! \times 1! \times 4}$$

$$= 15.$$

Ques:- Suppose A is a finite set with n element then number of element in largest equivalence relation is.

$$|A \times A| = n^2$$

Ques:- Number of function from m element set to n element set is

$$= n^m$$

6 WED

Ques:- The number of binary relation on a set with A with n element is

$$= 2^{n^2}$$

Ques:- Consider a binary relation $S = \{(x,y) \mid y = x+1,\ y, x \in \{0,1,2,3,\ldots\}\}$

the reflexive, transitive closure of S is

a) $\{(x,y) \mid y > x$ and $x,y \in \{0,1,2,\ldots\}\}$

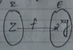
b) $\{(x,y) \mid y \geq x$ & $x,y \in \{0,1,2,\ldots\}\}$

c) $\{(x,y) \mid y \leq x$ & $x,y \in \{0,1,2,\ldots\}\}$

d) $\{(x,y) \mid y < x$ & $x,y \in \{0,1,2,\ldots\}\}$.

Notes

Ques:- let X, Y, Z be the set of size $|X|, |Y|, |Z|$, let $w = X \times Y$ and E be the set of all subsets of w, the number of function from Z to E is



$= 2^{xyz}$

Ques:- For the set N of natural numbers a binary operation $f: N \times N \rightarrow N$ an element Z belongs to N is called identity element if $f(a,z) = f(z,a) = a$. for all $a \in N$ then which of the following operations does not have an identity element?

a) $f(x,y) = x+y-3$.
b) $f(x,y) = \max(x,y)$
c) $f(x,y) = x^y$
d) N.O.T.

a) $x+e-3 = x$
$e+x-3 = x$] $e = 3 \in N$.

b) $e = 1$

c) $f(x,e) = x^e$
$f(e,x) = e^x$

$\therefore$ e does not exist.

Notes

Ques:- Let S be a set of n elements then the number of ordered pair in the largest and smallest equivalence relation on sare?

largest - n^2
Smallest - n.

Ques:- A Relation R is defined on ordered pair of integers as follows:-

(x,y) R (u,v) if $x<v$ and $y>v$

i) neither a partial order nor a equivalence relation.
ii) A partial order but not a equivalence relation.
iii) A total order
iv) equivalence relation.

Ques:- Consider a binary relation $R = \{(x,y), (x,z), (z,x), (z,y)\}$

a) R is symmetric but not antisymmetric.
b) R is not symmetric but antisymmetric.
c) R is both symmetric and anti symmetric.
d) R is neither symmetric nor antisymmetric.

Is it possible that a relation is both symmetric and antisymmetric:-
$\{(x,x), (y,y), (z,z)\}$

Notes

Sun	Mon	Tue	Wed	Thu	Fri	Sat
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

Que :- what is the possible number of reflexive relation on a set of 5 elements?

11 MON

$$2^{n^2-n}$$

Que :- Consider a set $S=\{1, \omega, \omega^2\}$, the set under multiplication operation is

1) Group
2) Ring
3) an integral domain
4) field.

Que :- Let A be a set of n elements, let C be a collection of distinct subset of A such that for any two subset S_1 and S_2 in C either $S_1 \subset S_2$ or $S_2 \subset S_1$.

12 TUE

what is the maximum Cardinality of C.

a) n ~~b) n+1~~ c) $2^{n-1}+1$ d) n!

$A=\{1,2\}$

$\{\emptyset, \{1\}, \{1,2\}\}$

$A=\{1,2,3\}$

$\{\emptyset, \{1\}, \{1,2\}, \{1,2,3\}\}$

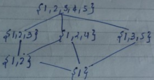
Notes

13 WED

Ques:- The inclusion of which of the following set into $\{\{1,2\}, \{1,2,3\}, \{1,3,5\}, \{1,2,4\}, \{1,2,3,4,5\}\}$ is necessary and sufficient to make S a complete latice under partial order relation by set containment.

(a) $\{1\}$ b) $\{1\}\ \{2,3\}$ c) $\{1\}, \{1,3\}$

d) $\{1\}\ \{1,3\}\ \{1,2,3,4\}\ \{1,2,3,5\}$.



14 THU

Ques:- Consider $S = \{a, b, c, d\}$. Consider the following four partition of S

$\pi_1 = \{\overline{abcd}\}$

$\pi_2 = \{\overline{ab}, \overline{cd}\}$

$\pi_3 = \{\overline{abc}, d\}$

$\pi_4 = \{\bar{a}, \bar{b}, \bar{c}, \bar{d}\}$

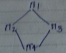
Let α be the partial order relation one the set of partitions $S = \{\pi_1, \pi_2, \pi_3, \pi_4\}$ defined as follows

$\pi_i \alpha \pi_j$ if π_i refines π_j.

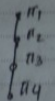
Notes

Thendraw Poset diagram for (S', α)

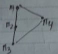
* π_j is a Refinement of π_i if all the element of π_j is a subset of some of the element of π_i



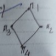
a]



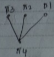
b)



c)



d)



let $S = \{1, 2, 3, \ldots, m\}$ such that $m > 3$ let $X_1, X_2, \ldots, X_n$ be the subset of S each of size 3. Define a function f from $S \to N$ as

$f(i)$ is the number of sets X_j that contain element i that is $f(i) = \{j \mid i \in X_j\}$

Notes

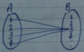
Then $\sum_{i=1}^{m} f(i)$ is

17 SUN

a) $3m$ b) $3n$ c) $2m+1$ d) $2n+1$

$\{1,2,3,4\}$

$\{1,2,3\}$, $\{1,2,4\}$, $\{2,3,4\}$, $\{1,3,4\}$.



$\{1,2,3,4,5\}$

$[\{1,2,3\}$ $\{1,2,4\}$, $\{1,2,5\}$, $\{2,3,4\}$, $\{2,3,5\}$, $\{3,4,5\}$

18 MON

$\{1,4,5\}$, $\{1,3,5\}$, $\{2,4,5\}$, $\{1,3,4\}]$

$m = 4$

$n = 10$

Notes

Linear Algebra
(Matrix)

A system of mn elements arranged in the form of rectangular array which has m rows and n columns is called matrix.

$A = [a_{ij}]_{m \times n}$.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Types of Matrix:-

1) Row Matrix:- A matrix in which elements are arranged in a single row however it has any no. of columns.

Ex:- $[a_{11} \ a_{12} \ a_{13} \ \cdots \ a_{1n}]_{1 \times n}$.

2) Column matrix:- A matrix in which elements are arranged in one Column.

Ex:- $\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{m1} \end{bmatrix}_{m \times 1}$

21 THU 3) Square matrix :- A matrix is said to be square if no. of rows = no. of columns (i.e $m = n$).

Ex:- $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$.

Diagonal element :- a_{ij} | $i=j$ $\{1,5,9\}$.

principal diagonal :- line along which diagonal element are present.

Off-diagonal element :- a_{ij} | $i \neq j$.

4) Diagonal Matrix :- a_{ij} | $i=j \neq 0$
$i \neq j = 0$

Square.

22 FRI $A = \text{diag}[1, 2, 3]$.

Properties of Diagonal matrix:-

1) $A = \text{dia}[1, 2, 3]$ $B = \text{diag}[4, 5, 6]$.

$A+B = \text{diag}[1+4, 2+5, 3+6]$.

2) $A = \text{diag}[a_{11}, a_{22}, a_{33}]$

$KA = \text{diag}[Ka_{11}, Ka_{22}, Ka_{33}]$.

Notes

3) $A = \text{diag}[a_{11}, a_{22}, a_{33}]$

$$A^n = \text{diag}[(a_{11})^n, (a_{22})^n, (a_{33})^n]$$

4) $A = \text{diag}[A_{11}, a_{22}, a_{33}]$

$$A^{-1} = [1/a_{11}, 1/a_{22}, 1/a_{33}].$$

1) Unit matrix
2) Upper Triangular
3) Lower Triangular.
4) Null matrix and its properties.
5) Involuntary Matrix : $A^2 = I$
6) Idempotent Matrix :- $A^2 = A$
7) Nilpotent Matrix :- $A^x = 0$, $A^{x-1} \neq 0$.

8) Addition of two matrix.

Equality b/w Matrices :-
Size equal and corresponding elements are also equal.

$$\begin{bmatrix} x-y & p+q \\ p-q & x+y \end{bmatrix} = \begin{bmatrix} 9 & 5 \\ 1 & 10 \end{bmatrix}.$$

25 MON

$$\begin{aligned} x - y &= 2 \\ x + y &= 10 \\ \hline 2x &= 12 \end{aligned} \Rightarrow x = 6.$$

$$\begin{aligned} p+q &= 5 \\ p-q &= 1 \\ \hline \Rightarrow 2p &= 6 \end{aligned} \Rightarrow p = 3.$$

$q = 5 - 3 = 2.$

Addition of two matrices :-

$A = a_{ij}]_{m \times n} \quad B = b_{ij}]_{m \times n}$

$A + B = [a_{ij} + b_{ij}]_{m \times n}$

1) $A+B$ is Commutative.
2) $A + (B+C) = (A+B) + C$
3) Additive identity = Null matrix

26 TUE

4) Additive inverse = $-A$.
5) $A + X = 0 \Rightarrow X = -A$.
6) $A + X = X + B \Rightarrow A = B$.

Multiplication of matrices by a Scalar.

1) K is a scalar and A is a matrix.

$K \cdot A = [K \cdot a_{ij}]_{m \times n}$.

2) $(p+q)A = pA + qA$.

3) $p(qA) = q(pA)$.

Notes

Matrix Multiplication :-

Two matrix can be multiplied if they are Conformable

$A = [a_{ij}]_{m \times n}$ $B = [b_{ij}]_{n \times p}$.

1) If $AB = 0 \Rightarrow A = 0$ or $B = 0$
$A \neq 0$ and $B \neq 0$

$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

$A \times B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

2) $AB = BA$ } → False.
⇒ Matrix multiplication is not Commutative.

3) $A \cdot (B \cdot C) = (A \cdot B) \cdot C$. [Applied that matrices should be Comparable]

4) $A \cdot (B + C) = A \cdot B + A \cdot C$.

If $A \cdot B = 0 \Rightarrow B \cdot A = 0$ → False.

Consider the matrices $X_{4 \times 3}$, $Y_{4 \times 3}$, $P_{2 \times 3}$.

The order of $[P(X^T Y)^{-1} P^T]^T$ will be?

Notes

29 FRI

$= [(2\times 3)\ (3\times 4)\cdot(4\times 3)\ (3\times 2)]^T$

$= [\ (2\times 4)\ (4\times 2)]^T = [(2\times 2)]^T = (2\times 2)$

Trace of a Matrix :-

Sum of diagonal elements.

$T = a_{11} + a_{22} + a_{33} + \ \cdot\ \cdot\ \cdot$

1) Trace $(\lambda\cdot A) = \lambda\cdot T$
2) Trace $(A+B) = T(A) + T(B)$
3) Trace $(A\cdot B) = $ Trace $(B\cdot A)$
when AB and BA defined

30 SAT

1) $(A')' = A$
2) $(A+B)' = A' + B'$
3) $(k\cdot A)' = k(A)'$
* 4) $(A\cdot B)' = B'\cdot A'$
* 5) $(A\cdot B\cdot C)' = C'\cdot B'\cdot A'$

Conjugate of a matrix :-

$A = \begin{bmatrix} 1+2i & 3 \\ 4 & 1-2j \end{bmatrix}$ $\quad \bar{A} = \begin{bmatrix} 1-2i & 3 \\ 4 & 1+2i \end{bmatrix}$

Notes

1) $\bar{\bar{A}} = A$
2) $\overline{(A+B)} = \bar{A} + \bar{B}$
3) $\overline{(K \cdot A)} = \bar{K} \cdot \bar{A}$ {If K is a Complex number}
4) $\overline{(A \cdot B)} = \bar{A} \cdot \bar{B}$.

Conjugate Transpose of a Matrix :-

$A^{\theta} : (\bar{A})^{T}$ or A^{*}

$$\begin{bmatrix} 1+i & 3 \\ 4 & 1-i \end{bmatrix}.$$

1) $(A^{\theta})^{\theta} = A$
2) $(A+B)^{\theta} = A^{\theta} + B^{\theta}$.
3) $(A \cdot B)^{\theta} = B^{\theta} \cdot A^{\theta}$
4) $(KA)^{\theta} = \bar{K}(A)^{\theta}$

Real Matrix :-

1) Symmetric if $A^{T} = A$
2) Skew Symmetric
3) orthogonal.

Symmetric :- $\begin{bmatrix} a & d & g \\ d & e & f \\ g & f & i \end{bmatrix}$

$A \cdot A^{T}$ is always Symmetric.

$(A \cdot A^{T})^{T} = (A^{T})^{T} \cdot (A)^{T}$

3 TUE

$= A \cdot A^T$

$\therefore$ $A \cdot A^T$ is always symmetric.

$\longrightarrow \frac{A+A^T}{2}$ is symmetric if A is symmetric or A is not symmetric doesn't matter.

$\frac{1}{2}[A+A^T]^T$

$= \frac{1}{2}[A^T + (A^T)^T] = \frac{1}{2}[A^T + A]$

$= \frac{1}{2}[A+A^T]$.

Skew symmetric Matrix :-

4 WED

$A^T = -A$.

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

Orthogonal Matrix :-

$A \cdot A^T = I$

$\Rightarrow A^T = A^{-1}$

$A \to D$
$A^T \to D$.

$|A| = ?$

$|A \cdot A^T| = |I|$

$|A|.|A^T|. = 1$.

$|A|^2 = 1 \Rightarrow |A| = \pm 1$

Notes: If matrices A and B are symmetric then matrices A+B and A-B are also symmetric.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
June 2012						1	2
	3	4	5	6	7	8	9
	10	11	12	13	14	15	16
	17	18	19	20	21	22	23
	24	25	26	27	28	29	30

Given an orthogonal Matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$.

5 THU

Then $(A \cdot A^T)^{-1}$ is

$$(A \cdot A^T)^{-1} = (I)^{-1} = I.$$

Real Matrix :-

Real Matrices $[A]_{3\times1}$ $[B]_{3\times3}$ $[C]_{3\times5}$ $[D]_{5\times3}$ $[E]_{5\times5}$ $[F]_{5\times1}$ are given. [B] and [E] are Symmetric.

Que:- following statements are made wrt these matrices -

1) $[F]^T [C]^T [B] [C] [F]$ is a Scalar (number)

2) $[D]^T [F] [D]$ is always Symmetric.

6 FRI

2) $\rightarrow (3\times5) \times (5\times1) \times (5\times3)$

$(3\times1) \times (5\times3) \rightarrow$ multiplication is not possible.

Que:- A Square matrix is skew symmetric if

1) $A^T = A$

2) $A^{-1} = A$

3) $A^T = -A$

4) $A^T = A^{-1}$

Notes

7 SAT

A. Singular Matrix 3
B. Non-singular Matrix
C. Real Matrix 4
D. orthogonal matrix 2
E - Non square matrix 1

1 Determinent is not defined!
2 " " always 1
3 " " " zero
4 Eigen values are real
5. " " not defined

1) The value of a determinant does not change when rows and columns are interchanged $\rightarrow$ True

2) If any Row or Column of a matrix is completely zero then determinant of that matrix is zero $\rightarrow$ true

8 SUN

3) If two rows or two columns are identical then determinant of that matrix is zero. $\rightarrow$ True.

4) If all elements of one row is multiplied by K then determinant is $K.|A|$. $\rightarrow$ True.

5) If A be a n-rows square matrix and K is any scalar then

$$|KA| = K^n . |A|$$

6) $|A.B| = |A|.|B|$

Notes

7) $|A^{-1}| = \frac{1}{|A|}$

8) $|A^n| = (|A|)^n$

→ Inverse, co-factor, adjoint
→ RANK
→ System of linear equation
→ Eigen value and eigen vector
→ Carley Hamilton theorem

1) Find inverse of $\begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$, $A^{-1} = ?$

$Adj(A) \cdot \dfrac{\begin{bmatrix} 7 & -5 \\ -2 & 1 \end{bmatrix}^T}{-3} = \dfrac{-1}{3}\begin{bmatrix} 7 & -2 \\ -5 & 1 \end{bmatrix}$.

$= \dfrac{1}{3}\begin{bmatrix} -7 & 2 \\ 5 & -1 \end{bmatrix}$.

2) The product of Matrix $(PQ)^{-1}P$

$= Q^{-1}P^{-1} \cdot P = Q^{-1} \cdot I$

$= Q^{-1}$

Notes

3) Find inverse of :-

$$\begin{bmatrix} 3+2i & i \\ -i & 3-2i \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$$

Ques :-

$$M = \begin{bmatrix} 3/5 & 4/5 \\ x & 3/5 \end{bmatrix}.$$

Transpose of $M = M^{-1}$. The value of x is.

ORthogonal

$$MM^T = I$$

$$\begin{bmatrix} 3/5 & 4/5 \\ x & 3/5 \end{bmatrix}\begin{bmatrix} 3/5 & x \\ 4/5 & 3/5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{3}{5}x + \frac{12}{25} = 0$$

$$\rightarrow x = -\frac{4}{5}.$$

Ques :- If $R = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 2 & 3 & 2 \end{bmatrix}$. Then top row of R^{-1} is.

$$A^{-1} = \frac{\operatorname{adj} A}{|A|}$$

$$\operatorname{adj} A = \begin{bmatrix} 5 & -6 & 4 \\ -3 & +4 & -3 \\ 1 & -1 & 1 \end{bmatrix}^T = \begin{bmatrix} 5 & -3 & 1 \\ -6 & 4 & -1 \\ 4 & -3 & 1 \end{bmatrix}$$

Notes

$|A| = 1(5) - 1(4) = 1.$

Que :- Let $A = \begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} 1/2 & a \\ 0 & b \end{bmatrix}$$

find a+b.

$$\begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1/2 & a \\ 0 & b \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2a - 0.1b \\ 0 & 3b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow b = 1/3$

$2a - \frac{0.1}{3} = 0 \quad a = \frac{0.1}{60}$

$a+b = \frac{1}{3} + \frac{1}{60} = \left(\frac{20+1}{60}\right) = \frac{21}{60}$ 20

$= \frac{7}{20}.$

Notes

Sun	Mon	Tue	Wed	Thu	Fri	Sat
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

August 2012

15 SUN

Ques :- Let ABCD be $n \times n$ matrices with non-zero determinant if $ABCD = I$ then B^{-1} is

a) ADC b) $D^{-1}C^{-1}A^{-1}$ c) CDA D) none

$$ABCD = I$$
$$BCD = A^{-1}$$
$$BC = A^{-1}D^{-1}$$
$$B = A^{-1}D^{-1}C^{-1}.$$

$$B \cdot B^{-1} = I.$$
$$A^{-1}D^{-1}C^{-1}B^{-1} = I \Rightarrow B^{-1} = CDA.$$

* Rank of a Matrix :-

16 MON

→ No. of non-zero rows.
→ No. of independent rows
→ It is defined for square and rectangular matrix

for $[A]_{m \times n}$

$$RANK(A) \leq \min(m, n).$$

$$= \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}_{3 \times 4}.$$

Notes

$$\begin{bmatrix} 4 & 7 & 1 \\ 6 & 3 & 4 \\ 2 & 1 & 0 \end{bmatrix}$$

$$= 1(0) - 4(0) + 0(12-12) = 0$$

$$\begin{bmatrix} 2 & 1 & 3 \\ 3 & 4 & 7 \\ 1 & 0 & 1 \end{bmatrix} = 1(4) + 4(-1) + 0(14-6)$$
$$= 0.$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = 8 - 3 = 5 \neq 0.$$

$\therefore$ RANK = 2.

System of Non-Homogeneous linear equation

$$4y + 3z = 8$$
$$2x - z = 2$$
$$3x + 2y = 5.$$

$AX = B.$

$$\left[\begin{array}{ccc|c} 0 & 4 & 3 & 8 \\ 2 & 0 & -1 & 2 \\ 3 & 2 & 0 & 5 \end{array}\right].$$

Notes

19 THU

$$= \begin{vmatrix} 2 & 0 & -1 & 2 \\ 0 & 4 & 3 & 8 \\ 3 & 2 & 0 & 5 \end{vmatrix} = \begin{bmatrix} 1 & 2 & -2 & 5 \\ 0 & 4 & 3 & 8 \\ 3 & 2 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & -2 & 5 \\ 0 & 4 & 3 & 8 \\ 0 & -4 & 6 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -2 & 5 \\ 0 & 4 & 3 & 8 \\ 0 & 0 & 9 & 18 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 5 \\ 0 & 2 & 3 & 8 \\ 0 & 0 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1/2 & 1/2 & -1 & 5/2 \\ 0 & 2 & 3 & 8 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 1/2 & -1 & 5/2 \\ 0 & 2 & 3 & 8 \\ 0 & 0 & 0 & 2+5/2 \end{bmatrix}.$$

$R(A) = 2$

$R(A/B) = 3$

20 FRI

$\therefore$ Inconsistent Solution.

Notes

$x + y + z = 5$
$x + 3y + 3z = 9$
$x + 2y + \alpha z = \beta$.

for what value of α and β above equation have an infinite no. of Solution.

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 1 & 3 & 3 & 9 \\ 1 & 2 & \alpha & \beta \end{array}\right] \quad R_2 = R_2 - 1 \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 2 & 2 & 4 \\ 1 & 2 & \alpha & \beta \end{array}\right]$$

$$R_3 = R_3 - R_1 \quad = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 2 & 2 & 4 \\ 0 & 1 & \alpha-1 & \beta-5 \end{array}\right] \quad R_3 = R_3 - \frac{1}{2}R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 2 & 2 & 4 \\ 0 & 0 & \alpha-2 & \beta-7 \end{array}\right]$$

$\alpha - 2 = 0$
$\beta - 7 = 0$
$\Rightarrow \alpha = 2$
$\Rightarrow \beta = 7$

Ques :-

$x + y + z = 3$
$x + 2y + 3z = 4$
$x + 4y + kz = 6$.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & k & 6 \end{array}\right]$$

Notes

23 MON For what value of K, system will not have a unique solution.

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & K-1 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & K-7 & 0 \end{bmatrix}$$

$K=7$, the system will not have have a unique solution.

Ques:- Consider the following system of linear equations 2nd and 3rd columns are linearly dependant.

24 TUE

$$\begin{bmatrix} 2 & 1 & -4 \\ 4 & 3 & -12 \\ 1 & 2 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \alpha \\ 5 \\ 7 \end{bmatrix}$$

For what or how many values of α the system will have infinite number of solutions.

$$\left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 4 & 3 & -12 & 5 \\ 1 & 2 & -8 & 7 \end{array}\right]$$

$$\begin{bmatrix} 2 & 1 & 2 & \alpha \\ 4 & 3 & 6 & 5 \\ 1 & 2 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 & \alpha \\ 4 & 3 & 6 & 5 \\ 0 & 0 & 0 & 7-2\alpha \end{bmatrix} \ldots$$

Notes

$R_3 = R_3 - \frac{1}{2} R_1$
$R_2 = R_2 - \frac{3}{2} R_1$

$$\begin{bmatrix} 2 & 1 & 4 & \alpha \\ 0 & 1 & -4 & 5-2\alpha \\ 0 & \frac{3}{2} & -6 & 7-\alpha/2 \end{bmatrix} \quad R_3 = R_3 - \frac{3}{2} R_1$$

Ques :-

$x_1 + 2x_2 + x_3 + 4x_4 = 2$
$3x_1 + 6x_2 + 3x_3 + 12x_4 = 6.$

which of the following statement is true?

1) Only trivial sol. $x_1 = x_2 = x_3 = x_4 = 0$.
2) No solution
3) A unique non trivial sol. exist.
4) Multiple non trivial sol exists.

$$\begin{bmatrix} 1 & 2 & 1 & 4 & 2 \\ 3 & 6 & 3 & 12 & 6 \end{bmatrix}$$

$R(A) = R(A/B) = 1 < 4$

Notes

27 FRI

CS-2008

$$x_1 + x_2 + 2x_3 = 1$$
$$x_1 + 2x_2 + 3y_3 = 2$$
$$x_1 + 4x_2 + ax_3 = 4.$$

The above system of equation has a unique solution for how many values of a?

$$= \left| \begin{matrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 1 & 4 & a & 4 \end{matrix} \right|$$

$$= \left| \begin{matrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & a-2 & 3 \end{matrix} \right| = \left| \begin{matrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & a-5 & 0 \end{matrix} \right|$$

28 SAT

$$a - 5 \neq 0 \Rightarrow a \neq 5.$$

Ques:-

$$A = \begin{bmatrix} 4 & -2 \\ +2 & 1 \end{bmatrix}$$

$$|A - \lambda I| = 0.$$

$$\begin{bmatrix} 4 & -2 \\ +2 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 4-\lambda & -2 \\ 2 & 1-\lambda \end{bmatrix} = 0.$$

Notes

$(4-\lambda)(1-\lambda) - (-2\times2) = 0$ → Characteristic equation

$\lambda = ?$ → eigenvalues

$$P = \begin{bmatrix} 4 & 5 \\ 2 & -5 \end{bmatrix}$$

$$\begin{vmatrix} 4-\lambda & 5 \\ 2 & -5-\lambda \end{vmatrix} = 0$$

$(4-\lambda)(5-\lambda) - 5\times2 = 0$

$= 20 - 4\lambda + 5\lambda + \lambda^2 = 10$

$\Rightarrow \lambda + \lambda^2 - 30 = 0$

$\lambda^2 - 6\lambda + 5\lambda - 30 = 0$

$\lambda^2 - \lambda - 30 = 0$

$\Rightarrow \lambda(\lambda-6) + 5(\lambda-6) = 0$

$\Rightarrow \lambda = 6, -5$

Note :- Sum of eigen values = Trace of the Matrix
Product of eigenvalues = Determinant of the given matrix.

Eigenvalues are possible for square matrix as determinant is possible only for square matrix.

Notes

Eigen vectors :-

$[A - \lambda I][X] = 0$

eigen value of A = ?

$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

$(2-\lambda)(2-\lambda) - 0 = 0$

$\Rightarrow \lambda = 2$

for $\lambda = 2$

$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$0x_1 + x_2 = 0 \quad \Rightarrow x_2 = 0$

$0x_1 + 0x_2 = 0 \quad . \; x_1 = ? = K$

for $\lambda \rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} K \\ 0 \end{bmatrix}$

possible answers :- $\begin{bmatrix} 5 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$

Note :- $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 20 \\ 40 \end{bmatrix}$

ratio must remain Same

Notes

→ Distinct number of eigenvalues = Distinct number of eigen vectors.

Ques:- $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ eigen vectors of given matrices are ?

$\begin{bmatrix} 1 \\ a \end{bmatrix}$ & $\begin{bmatrix} 1 \\ b \end{bmatrix}$ find sum of $(a+b)$?

$$\begin{vmatrix} 1-\lambda & 2 \\ 0 & 2-\lambda \end{vmatrix} = 0.$$

$(1-\lambda)(2-\lambda) = 0$

$2 - \lambda + 2\lambda(-1) + \lambda^2 = 0.$

$\Rightarrow \lambda^2 - 3\lambda + 2 = 0$

$\lambda^2 - 2\lambda - \lambda + 2 = 0.$

$\lambda(\lambda-2) - 1(\lambda-2) = 0 \Rightarrow \lambda = 2, 1$

$\lambda = 2$ $\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-x_1 + 2y_1 = 0$ for $x_1 = 1$, $y_1 = 1/2$

$\lambda = 1$ $\begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2y_1 = 0$

$y_1 = 0$

$\begin{bmatrix} x_1 \\ 0 \end{bmatrix}$

Notes

3 FRI $\begin{bmatrix}1\\0\end{bmatrix}$, $\begin{bmatrix}1\\1/2\end{bmatrix}$

$a = 0, \quad b = 1/2$

$\Rightarrow a+b = 1/2$

Ques :-

$$\begin{bmatrix}5 & 0 & 0 & 0\\0 & 5 & 5 & 0\\0 & 0 & 2 & 1\\0 & 0 & 3 & 1\end{bmatrix}$$

a) $\begin{bmatrix}1\\-2\\0\\0\end{bmatrix}$ b) $\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$ c) $\begin{bmatrix}1\\0\\0\\-2\end{bmatrix}$ d) $\begin{bmatrix}1\\-1\\2\\1\end{bmatrix}$

4 SAT $(5-\lambda)\left[(5-\lambda)(2-3)\right] = 0$

$-(5-\lambda)(5-\lambda) = 0$

$\Rightarrow \lambda = 5$

$$\begin{bmatrix}0 & 0 & 0 & 0\\0 & 0 & 5 & 0\\0 & 0 & -3 & 1\\0 & 0 & 3 & -4\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$5x_3 = 0 \Rightarrow x_3 = 0$

$-3x_3 + x_4 = 0$

$3x_3 - 4x_4 = 0$

$x_4 = 0$

$\Rightarrow x_3 = 0$

$= \begin{bmatrix}k\\k\\0\\0\end{bmatrix}$

Notes

$\begin{bmatrix} 1 & -2 & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ the eigen values corresponding to eigen vector $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.

$(4-\lambda)(4-\lambda) - (4) = 0$.

$16 - 4\lambda - 4\lambda + \lambda^2 - 4 = 0$.

$\Rightarrow \lambda^2 - 8\lambda + 12 = 0$

$\lambda^2 - 6\lambda - 2\lambda + 12 = 0$.

$\lambda(\lambda - 6) - 2(\lambda - 6) = 0$

$\lambda = 6, 2$

$\lambda = 2$

$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2x_1 + 2x_2 = 0$.

$\lambda = 6$

$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-2x_1 + 2x_2 = 0$

$2x_1 - 2x_2 = 0$

for $x_1 = 1, x_2 = 1$

$2x_1 - 2x_2 = 0$.

$\Rightarrow \lambda = 6$.

Notes

7 TUE

Que :- How many of the following matrices have an eigen value = 1.

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$$

1) $(1-\lambda)0 - 0 = 0$. ✓

2) $\lambda^2 = 0 \rightarrow \lambda = 0$

3) $(1-\lambda)^2 + 1 = 0$

4) $(1+\lambda)^2 = 0$

* Eigen values of A and A^T are same
* Eigen values of upper triangular matrices or lower triangular matrices or diagonal matrices are diagonal elements itself.

8 WED

* If $\lambda_1, \lambda_2, \lambda_3 \dots \lambda_n$ are eigen values of [A] then eigen values of $K \cdot A = K\lambda_1, K\lambda_2, \dots, K\lambda_n$.

* If $\lambda_1, \lambda_2, \dots \lambda_n$ are eigen values of matrix A then eigen values of A^{-1} are $1/\lambda_1, 1/\lambda_2, 1/\lambda_3 \dots 1/\lambda_n$

* If $\lambda_1, \lambda_2, \dots \lambda_n$ are eigen values of matrix A then eigen values of matrix A^n are $\lambda_1^n, \lambda_2^n, \dots, \lambda_n^n$

* maximum number of distinct eigen values is equal to the size of the matrix.

Notes

* If an matrix A is of size $n \times n$ and it has n distinct eigen values then it will have n distinct or linearly independent eigen vectors.

1) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ are -2 and $+6$ find other?

$x + (-2) + 6 = 1 + 5 + 1$
$x + 4 = 7$
$\Rightarrow x = 3$

2) Eigen values of $S = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$. 5 and 1

then what are the eigen values of $S^2 = S \cdot S$.

25, 1

3) The eigen value of following matrix $\begin{bmatrix} -1 & 3 & 1 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$

a) $3, 3+5i, 6-i$

b) $3+i, 3-i, 5+i$

c) $-6+5i, 3+i, 3-i$

d) $3, -1+3i, -1-3i$

11 SAT To find higher power of A and also its inverse by CALEY HAMILTON THEOREM.

$$A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

Find eigen values of A?

$$(1-\lambda)(2-\lambda) - 12 = 0.$$
$$\lambda^2 - 3\lambda - 10 = 0.$$

Replace λ by A and const by C.C

$$A^2 - 3A - 10I = 0.$$

12 SUN

$$\Rightarrow A^2 = 3A + 10I$$

$$= 3\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 9 \\ 12 & 6 \end{bmatrix} + \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$A^4 = A^2 \cdot A^2$$
$$= (3A + 10I)^2$$
$$= 9A^2 + 100I + 60A.$$
$$= 9(3A + 10I) + 100I + 60A$$
$$= 27A + 90I + 100I + 60A.$$

Notes

$$A^3 = A^2 \cdot A = 3A^2 + 10A$$
$$= 3(3A+10) + 10A$$
$$= 9A^2 + 30 + 10A$$

A^{-1} ?

$$A^2 - 3A - 10I = 0$$
$$A^{-1}A^2 - 3A^{-1}A - 10A^{-1} \cdot I = 0$$
$$\Rightarrow A - 3I - 10A^{-1} = 0$$
$$\Rightarrow 10A^{-1} = A - 3I$$
$$\Rightarrow A^{-1} = \frac{A-3I}{10}$$

A^9: $\begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$

Ans :- $511A + 510I$

$$(-3-\lambda)(0-\lambda) + 2 = 0$$
$$3\lambda + \lambda^2 + 2 = 0$$
$$\lambda^2 + 3\lambda + 2 = 0$$
$$A^2 + 3A + 2I = 0$$
$$\Rightarrow A^2 = -(3A + 2I)$$

$$A^4 = (3A+2I)^2$$
$$= 9A^2 + 4I^2 + 12A$$
$$= 9(-3A \mp 2I) + 4I^2 + 12A$$
$$= -27A \mp 18I + 4I^2 + 12A$$
$$= -15A \mp 14I$$

$$A^4 \cdot A^4 = (-15A \mp 14I)^2$$

Notes

$= 225A^2 + \cancel{144}^{196} I^2 + 15A \times \cancel{24}^{14} \times 2$

15 WED

$= 225(-3A - 2I) + 196I + 420A$

$= -675A - 450I + 196I + 420A$

$= -255A - 254I$

$A^8 \cdot A = (-255A - 254I) \cdot A$

$A^9 = (-255A^2 - 254A)$

$A^9 = -255(-3A - 2I) - 254A$

$A^9 = 765A + 510I - 254A$

$\therefore A^9 = 511A + 510I$

Ques:-

What are the eigen values for:- $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}$

16 THU

$$\begin{vmatrix} 5-\lambda & 0 & 0 & 0 \\ 0 & 5-\lambda & 5 & 0 \\ 0 & 0 & 2-\lambda & 1 \\ 0 & 0 & 3 & 1-\lambda \end{vmatrix} = 0$$

$(5-\lambda)\left[(5-\lambda)\left[(2-\lambda)(1-\lambda) - 3\right]\right] = 0.$

$\lambda = 5, 5.$

$(2-\lambda)(1-\lambda) - 3 = 0$

$\Rightarrow 2 - 2\lambda - \lambda + \lambda^2 = 3.$

$\Rightarrow \lambda^2 - 3\lambda - 1 = 0.$

$\Rightarrow \lambda = \left(\dfrac{3 \pm \sqrt{13}}{2}\right)$

Notes

* The characteristic roots or eigen values of a Hermition matrix are real.
  for Hermition matrix $A^\theta = A$.

* The characteristic roots of real Symmetric matrix are real.

* The characteristic roots of skew Hermition matrix are pure imaginary or zero.

  skew Hermition matrix :- $A^\theta = -A$

* The characteristic roots of skew symmetric matrix are either zero or pure imaginary.

* characteristic roots of unitary matrix are of unit modulus.

unitary matrix :- $A \cdot A^\theta = I$

$|\lambda| = 1$

$\Rightarrow \lambda = \pm 1$

* The characteristic roots of orthogonal matrix is also of unit modulus.

* If X is a characteristic vector of matrix A corresponding to characteristic value λ then $K.X$ is also a characteristic vector of A corresponding to same eigenvalue λ.

Notes

19 SUN

Ques :-

The eigen values and corresponding eigen vectors of a 2×2 matrix are given by

$\lambda_1 = 8$ $\quad V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad V_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

$\lambda_2 = 4$

The matrix is ?

a) $\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$ b) $\begin{bmatrix} 4 & 6 \\ 6 & 4 \end{bmatrix}$

c) $\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 4 & 8 \\ 8 & 4 \end{bmatrix}$.

20 MON

Ques :- $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$ all entries are non-zeros and one of its eigen values are zero then which of the following is true?

a) $P_{11}P_{22} - P_{12}P_{21} = 1$ b) $P_{11}P_{22} - P_{12}P_{21} = 0$

c) $P_{11}P_{22} - P_{12}P_{21} = -1$. d) $P_{11}P_{22} + P_{21} + P_{12} = 0$

Notes

Ques :- The trace and determinant of a 2×2 matrix are known to be −2 and −35. Its eigen values are.

a) −30 and −5

b) −37 and −1

c) −7 and 5

d) 17.5 and −2

Ques :- One of the eigen values of the given matrix is 3. then sum of other two is ?

$p+1-3 = p-2$

Ques :- $A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$ if eigen values of A are 4 and 8 then

(Gate 2010 - CS)

a) $x = 4, y = 10$

b) $x = 5, y = 8$

c) $x = -3, y = 9$

d) $x = -4, y = 10$

1) $2 + y = 8 + 4$

$\Rightarrow y = 10$.

2) $2y - 3x = 32$

$20 - 32 = 3x$

$\Rightarrow x = \frac{12}{3} = -4$

Ques :- $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 3 \end{bmatrix}$

eigen values = ?

Notes

$= 1, 4, 3.$

23 THU

Similar Matrix :-

A and B are said to be similar if there exists a non - singular matrix M such that

$$B = M^{-1} \cdot A \cdot M$$

then A and B are similar matrix also B is a diagonal matrix whose diagonal element are eigenvalues of A.

→ M is a matrix in which its columns are eigenvectors of A.

$$\lambda_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \lambda_2 = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$$

24 FRI

$$M = \begin{bmatrix} x_1 & x_3 \\ x_2 & x_4 \end{bmatrix}.$$

→ The Above process is called diagonalization

Ques :- $A = \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}.$

$$\begin{vmatrix} 4-\lambda & -2 \\ -2 & 1-\lambda \end{vmatrix} = 0.$$

$$(4-\lambda)(1-\lambda) - 4 = 0.$$

$$4 - 4\lambda - \lambda + \lambda^2 - 4 = 0.$$

$$\Rightarrow \lambda^2 - 5\lambda = 0 \quad \Rightarrow \lambda(\lambda-5) = 0 \Rightarrow \lambda = 0, 5.$$

Notes

for $\lambda = 0$.

$$\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$4x_1 - 2x_2 = 0$ $\Rightarrow$ $x_2 = 2x_1$

$-2x_1 + x_2 = 0$

$-4x_1 + 2x_2 = 0$

$\begin{bmatrix} K \\ 2K \end{bmatrix}$. $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

for $\lambda = 5$

$$\begin{bmatrix} -1 & -2 \\ -2 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$-x_1 - 2x_2 = 0$

$-2x_1 - 4x_2 = 0$

$\Rightarrow \frac{x_1}{x_2} = -2.$

$\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$M = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

$M^{-1} = \frac{1}{-5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Notes

$= \begin{bmatrix} +1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}.$

27 MON

$$= \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ \frac{8}{5}+\frac{2}{5} & -\frac{4}{5}-\frac{1}{5} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 4+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$$

* Similar matrices have same eigen values.

28 TUE

eigen values for A and A^T are same.

$$B = M^{-1}AM$$

$$MB = MM^{-1}AM = AM$$

$$MBM^{-1} = AMM^{-1} = A$$

$\Rightarrow A = MBM^{-1}$ $M \to$ eigen vectors of A.

$A^2 = A \cdot A = (MBM^{-1})(MBM^{-1})$, MB^2M^{-1}

$A^3 = ?$

$A^5 = M \cdot B^5 \cdot M^{-1}$

$A^{10} = M \cdot \underbrace{B^{10}}_{\text{Diagonal matrix.}} \cdot M^{-1}$

Notes

i) $AX = b \rightarrow$ Non-Homogeneous matrix
ii) $AX = 0 \rightarrow$ Homogeneous equation

$$[A_{m\times n}][x]_{n\times 1} = [b]_{m\times 1}$$

$m \rightarrow$ equations
$n \rightarrow$ variables

1) $x + 3y - 2z = 0$
$2x - y + 4z = 0$
$x - 11y + 14z = 0$

1) $x = 0$ and $y = 0$ and $z = 0$ (trivial Solution)
2) Non-trivial Solution.

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & 8 \\ 0 & -14 & 16 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ Rank(A) = 2.

(i) $-7y + 8z = 0$ $\Rightarrow$ Let $z = c$.
$-7y = -8c$
(ii) $x + 3y - 2z = 0$ $\Rightarrow y = \frac{8}{7}c$

Notes

$$x + \frac{3\cdot 8}{7}c - 2c = 0$$

$$\Rightarrow x = -\frac{10}{7}c$$

i) only trivial Solution where $R(A)$ = no. of variables
$x = 0$ and $y = 0$ and $z = 0$

ii) $R(A) <$ no. of variables.
Infinitely many solutions are possible.

$$= \begin{bmatrix} 1 & a & a^2 & \cdots & a^n \\ 1 & a & a^2 & \cdots & a^n \\ \vdots \\ 1 & a & a^2 & \cdots & a^n \end{bmatrix} \quad \begin{array}{l} R_2 = R_2 - R_1 \\ R_3 = R_3 - R_1 \\ \vdots \\ R_n = R_n - R_1 \end{array}$$

$\therefore$ Rank = 1

2) $$A = \begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad \begin{array}{l} |A| = 6 \times 2 \times 4 \times -1 \\ \Rightarrow |A| = -48. \end{array}$$

3) $$\begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & -2 & 0 \end{vmatrix} = -1(-1) = +1$$

Ans $= -1(|B|) = -1(+1) = -1$.

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 7 & 2 \\ 2 & 0 & 2 & 0 \\ 9 & 0 & 6 & 1 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 1 & 7 & 2 \\ 0 & 2 & 0 \\ 0 & 6 & 1 \end{vmatrix} = 1(2) = 2$$

$$2 \times (|B|) = 2 \times 2 = 4.$$

Que:- Let A be a 2×2 matrix such that $a_{11} = a_{12} = a_{21} = +1$ and $a_{22} = -1$. Find eigen values of A^{18}.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

3 MON

$A^8 = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$

$A \begin{cases} \lambda_1 = +\sqrt{2} \\ \lambda_2 = -\sqrt{2} \end{cases}$

$A^{14} = (\sqrt{2})^{14} = 64\sqrt{2}$
$(-\sqrt{2})^{14} = -64\sqrt{2}$

$(\sqrt{2})^2 = 2$
$(\sqrt{2})^4 = 4$
$(\sqrt{2})^8 = 16$
$(\sqrt{2})^{16} = 256$

Que:- If A and B are real Symmetric matrices of size n×n then

a) $A \cdot A^t = I$
b) $A = A^{-1}$
c) $A \cdot B = BA$
d) $(AB)^t = BA$

$(A)^t = A$
$(B)^t = B$

4 TUE

$(A \cdot B)^t = B^t A^t = B \cdot A$

Que:- $P = \begin{bmatrix} a & 1 & 0 \\ 1 & a & 1 \\ 0 & 1 & a \end{bmatrix}$ find e.v.

a) $a, a-\sqrt{2}, a+\sqrt{2}$
b) a, a, a
c) $a, a, 2a$
d) $-a, 2a, 2a$

$|P| = a(a^2-1) - 1(a)$
$= a^3 - a - a$
$= a^3 - 2a$

Note:

Numerical Methods :-

5 WED

1) Solving system of linear equation by
a) triangularization / factorization / LU decomposition method.
b) Gauss elimination method by partial pivoting.
c) Jacobi method.
d) Gauss Seidal method.

ii) Solving $f(x) = 0$ by.
a) Bisection method.
b) Regula falsi method.
c) Secant method.
d) Newton - Raphson method.

iii) Numerical interpretation by.
i) Trapezoidal Rule.
ii) Simpson Rule.

6 THU

i) Triangularization / factorization / LU dec. method :-

$AX = B$

$\downarrow$

factorization coefficient matrix.

Condition :- If all principle minor of A is non-zero

Principle minor :-

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

$1 \times 1 : [1] = 1$

$2 \times 2 : \begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix} = -3.$

$3 \times 3 : \begin{vmatrix} 1 & 4 & 7 \\ 3 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix} = 0.$

Notes

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \times \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$u_{11} = 2,\ u_{12} = 3,\ u_{13} = 1.$

$u_{11} \times l_{21} = 1$

$\Rightarrow l_{21} = 1/2$

$u_{12} \times l_{21} + u_{22} = 2$

$\frac{u_{12}}{2} + u_{22} = 2.$

$\Rightarrow u_{22} = 2 - \frac{3}{2} = \frac{1}{2}.$

$u_{13} l_{21} + u_{23} = 3.$

$1 \times \frac{1}{2} + u_{23} = 3.$

$\Rightarrow u_{23} = 3 - 1/2 = 5/2.$

$u_{31} \times u_{11} = 3.$

$\Rightarrow u_{31} = 3/2$

$l_{31} \times u_{12} + l_{32} \times u_{22} = 1$

$3 l_{31} + \frac{1}{2} l_{32} = 1$

$l_{31} \times u_{13} + l_{32} \times u_{23} + u_{33} = 2.$

October 2012	Sun	Mon	Tue	Wed	Thu	Fri	Sat
		1	2	3	4	5	6
	7	8	9	10	11	12	13
	14	15	16	17	18	19	20
	21	22	23	24	25	26	27
	28	29	30	31			

11 TUE

On solving,

$u_{11} = 2,\ u_{12} = 3,\ u_{13} = 1,\ l_{21} = 1/2,\ l_{31} = 3/2,$
$l_{32} = -7,\ u_{22} = 1/2,\ u_{23} = 5/2,\ u_{33} = 18.$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix}.$$

$AX = B$

$LUX = B.$

$UX = Y$

$LY = B.$

12 WED

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}.$$

$y_1 = 9$ $\qquad$ $y_3 = 5.$

$\frac{1}{2} y_1 + 1 y_2 = 6.$

$\therefore y_2 = 3/2$

$UX = Y.$

$$\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3/2 \\ 5 \end{bmatrix}$$

Notes

Sun	Mon	Tue	Wed	Thu	Fri	Sat
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

$18z = 5$

$\Rightarrow z = 5/18$

$\frac{1}{2}y + \frac{5}{2}z = 3/2$

$y = \frac{29}{18}$

$x = \frac{35}{8}$

2) Gauss elimination method using partial pivoting

$2x + y + z = 10$
$3x + 2y + 3z = 18$
$x + 4y + 9z = 16$

$$= \left[\begin{array}{ccc|c} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{array}\right]$$

$$\cdot \begin{bmatrix} 3 & 2 & 3 & 18 \\ 2 & 1 & 1 & 10 \\ 1 & 4 & 9 & 16 \end{bmatrix}$$

$R_3 = R_3 - \frac{1}{3}R_1$

$R_2 = R_2 - \frac{2}{3}R_1$

$$\cdot \begin{bmatrix} 3 & 2 & 3 & 18 \\ 0 & -1/3 & -1 & -2 \\ 0 & 10/3 & 8 & 10 \end{bmatrix}$$

Notes

15 SAT

$$= \begin{bmatrix} 3 & 2 & 3 & 18 \\ 0 & 10/3 & 8 & 10 \\ 0 & -1/3 & -1 & -2 \end{bmatrix} \quad R_3 = R_3 + \frac{1}{10} R_2 .$$

$$= \begin{bmatrix} 3 & 2 & 3 & 18 \\ 0 & 10/3 & 8 & 10 \\ 0 & 0 & -1/5 & -1 \end{bmatrix}$$

$$-\frac{1}{5} z = -1$$

$$\Rightarrow z = 5.$$

$$\frac{10}{3} y + 8z = 10 \quad \Rightarrow y = -9$$

16 SUN

$$3x + 2y + 3z = 18$$

$$\Rightarrow x = 7$$

1) Forward Pass — x and y are eliminated.
2) Backward Substitution
3) Pivot element.

<u>Ques</u> :-

$$5x + y + 2z = 34$$

$$4y - 3z = 12$$

$$10x - 2y + z = -4$$

In the solution of above set of linear equations by Gauss elimination using partial pivoting. The pivots for elimination of x and y are)

$$\begin{bmatrix} 5 & 1 & 2 & 34 \\ 0 & 4 & -3 & 12 \\ 10 & -2 & 1 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -2 & 1 & -4 \\ 0 & 4 & -3 & 12 \\ 5 & 1 & 2 & 34 \end{bmatrix} \quad R_3 = R_3 - \frac{1}{2} R_1$$

$$= \begin{bmatrix} 10 & -2 & 1 & -4 \\ 0 & 4 & -3 & 12 \\ 0 & 2 & 3/2 & 36 \end{bmatrix} \quad R_3 = R_3 - \frac{1}{2} R_2$$

$$= \begin{bmatrix} 10 & -2 & 1 & -4 \\ 0 & 4 & -3 & 12 \\ 0 & 0 & \cancel{6/2}\,3 & 30 \end{bmatrix}$$

$\therefore$ Pivot elements :- 10, 4.

Iterative method $\rightarrow$ JACOBI
$\rightarrow$ G. seidal.

JACOBI :-

$x = 0, y = 0, z = 0 \rightarrow$ initial value.

19 WED

Ques :-

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

I) $2x = 10 - y - z$

$\Rightarrow x = (10 - y - z)/2$

II) $2y = 18 - 3x - 3z$

$\Rightarrow y = \left(\dfrac{18 - 3x - 3z}{2}\right)$

III) $9z = 16 - x - 4y$

20 THU

JACOBI :-

(uses value of previous step).

I No.	x	y	z
0	0	0	0
1	5	9	16/9
2			
3			

G.S :-

	x	y	z
0	0	0	0
1	5	3/2	5/9
2			
3			
4			

Notes

Sun	Mon	Tue	Wed	Thu	Fri	Sat
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

Jacobi

$$x = \left(10 - 9 - \frac{16}{9}\right)/2$$

$$= (90 - 81 - 16)/18$$

$$= -7/18$$

$$y = \left(18 - 3(5) - 3(16/9)\right)/2$$

$$= \left(18 - 15 - 16/3\right)/2$$

$$= \left(3 - 16/3\right)/2$$

$$= -7/6$$

* Convergence rate of Gauss Seidal method is higher than JACOBI.

Matrix of G.S. Method:-

$$(L+D)X^{i+1} = -UX^{i} + B.$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}. \qquad L = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 7 & 8 & 0 \end{bmatrix}$$

Notes

23 SUN

$$U = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$x^{i+1} = (L+D)^{-1} \times (-U) x^{i} + (L+D)^{-1} \times B$$

Let $(L+D)^{-1} \times (-U) = H$ and $(L+D)^{-1} \cdot B = B'$

Then $x^{i+1} = Hx^{i} + B'$

(i) find eigen values of H and find its modulus.
(ii) Then find maximum eigen value.

(i) $\rho(H) < 1 \rightarrow$ Converge
(ii) $\rho(H) \geq 1 \rightarrow$ Diverge

24 MON

where $\rho(H)$:- is the maximum eigen value.

$$2x + y = 5 \quad \text{(i)}$$
$$3x - y = 2 \quad \text{(ii)}$$

$$A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix} \qquad L = \begin{bmatrix} 0 & 0 \\ 3 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Notes

$$H = (L+D)^{-1} \times (-U)$$

25 TUE

$$= \left(\begin{bmatrix} 0 & 0 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \right)^{-1} \times \left(- \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2 & 0 \\ 3 & -1 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} -1 & -3 \\ 0 & 2 \end{bmatrix}^{T} \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} -1 & 0 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ +3/2 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1/2 \\ 0 & -3/2 \end{bmatrix}$$

26 WED

$ev = 0, -3/2$

$= 0, 3/2$

$\rho(H) = 3/2$

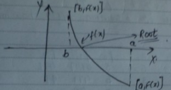
$3/2 > 1 \rightarrow$ diverge.

→ If the system is diagonally dominant then the system will converge.

Notes

Solve $f(x) = 0$.

1) Bisection method.



$f(x) = x^2 - 2 = 0$. $[1, 2]$

$\left(x_2 = \frac{x_1 + x_0}{2}\right)$

for $f_2 < 0, 7$ fix f_2

S.No	x_0	x_1	x_2	f_0	f_1	f_2
1	1	2	1.5	-1	2	0.25
2	1	1.5	1.25	-1	0.25	-0.42
3.	1.25	1.5	1.375	-0.42	0.25	-0.13412
4.	1.375	1.5	1.4375	-0.10937	0.25	0.06641
5	1.375	1.4375	$\boxed{1.40625}$	-0.10937	0.06641	-0.02246

$$x_{i+1} = \left(\frac{x_i + x_{i-1}}{2}\right)$$

→ Two guess values are required for x_0 and x_1, Solution is guaranteed.

Notes

Sun	Mon	Tue	Wed	Thu	Fri	Sat
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

August 2012

Que:- The Bisection method is applied to compute zero of a function

29 SAT

$f(x) = x^4 - x^3 - x^2 - 4$ [1, 9].

The method converges to a solution after how many iterations.

S.No.	x_0	x_1	x_2	f_0	f_1	f_2
	1	9	5	-5	5747	471
	1	5	3	-5	471	41
	1	3	2	-5	41	0

∴ The method after third iteration converges to the solution.

Regula-falsi method :-

30 SUN

$f(x) = x^2 - 2 = 0$ [1, 2]

$$x_2 = \frac{f_1 x_0 - f_0 x_1}{f_1 - f_0}$$

S.No.	x_0	x_1	x_2	f_0	f_1	f_2
	1	2	4/3	-1	2	-2/9
	4/3	2	7/5	-2/9	2	-1/25
	7/5	2				

Notes

1 MON

Ist :

$$= \left(\frac{2\times 1 - (-1)\times 2}{3}\right) = \frac{4}{3}.$$

$$f(4/3) = \frac{16}{9} - 2 = \frac{-2}{9}$$

$$\left\{\begin{array}{l} f_0 \cdot f_2 < 0 \Rightarrow \\ \quad x_1 \leftarrow x_2 \\ \text{or} \\ f_1 \cdot f_2 < 0 \Rightarrow \\ \quad x_0 \leftarrow x_2 \end{array}\right\}$$

IInd :-

$$\left(\frac{2\times\frac{4}{3} + \frac{2}{9}\times 2}{2+\frac{2}{9}}\right) = \frac{\frac{8}{3}+\frac{4}{9}}{\frac{20}{9}} = \frac{\frac{28}{9}}{\frac{20}{9}}$$

$$= \frac{\cancel{28}\,\cancel{4}\ 7}{\cancel{20}\,\cancel{4}\ 5}$$

2 TUE

$$f(7/5) = \frac{49}{25} - 2 = -1/25$$

Secant method :-

$$x_2 = \left(\frac{f_1 x_0 - f_0 x_1}{f_1 - f_0}\right)$$

$x_0 \leftarrow x_1$ (x_0 is replaced by x_1) and $x_1 \leftarrow x_2$

$f(x) = x^2 - 2 = 0 \quad [1, 2]$.

S.No.	x_0	x_1	x_2	f_0	f_1	f_2
1	1	2	4/3	-1	2	-2/9
2	2	4/3	7/5	2	-2/9	-1/25
3	4/3	7/5				

Newton-Raphson method :-

→ only one guess value is required.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f(x) = x^2 - 2 = 0$, $x_0 = 1$

$$x_1 = x - \left(\frac{x^2 - 2}{2x}\right)$$

$$= \frac{2x^2 - x^2 + 2}{2x} = \frac{x^2 + 2}{2x}$$

$$\therefore x_1 = \left(\frac{x^2 + 2}{2x}\right)$$

5 FRI

I No.	x_0	x_1
1	-	1
2	1	3/2
3	3/2	17/12

Find iterative equation for :-

$f(x) = e^x - \sin x = 0$.

$$= x - \left(\frac{e^x - \sin x}{e^x - \cos x}\right)$$

$$= \left[\frac{x(e^x - \cos x) - (e^x - \sin x)}{(e^x - \cos x)}\right].$$

$$= \left(\frac{xe^x - x\cos x - e^x + \sin x}{e^x - \cos x}\right).$$

6 SAT

Date-2010 :-

Newton raphson method :-

$f(x) : x^2 - 13 = 0$ with initial value as 3.5.

$f(x) = x^2 - 13$. $f'(x) = 2x$.

$$\left(x - \left(\frac{x^2 - 13}{2x}\right)\right)$$

$$= \left(\frac{2x^2 - x^2 + 13}{2x}\right) = \frac{x^2 + 13}{2x}$$

$= 3.607$.

Notes

$x = e^{-x}$ using NR method.

7 SUN

$\cancel{x - \frac{e^{-x}}{(-e^{-x})}}$ $\cancel{x + \frac{e^{-x}}{e^{-x}}}$ $\cancel{x+1}$

$f(x) = x - e^{-x} = 0$
$f'(x) = 1 + e^{-x}$

$$x - \frac{(x - e^{-x})}{(1+e^{-x})} = \left(\frac{x + xe^{-x} - x + e^{-x}}{1+e^{-x}}\right)$$

Que:-

$x^3 - x^2 + 4x - 4 = 0$

is to be solved using newton Raphson method if $x = 2$ is taken as the initial approximation of solution then the next approximation is

8 MON

$f(x) = x^3 - x^2 + 4x - 4$
$f'(x) = 3x^2 - 2x + 4$

$$\left[x_1 - \left(\frac{x^3 - x^2 + 4x - 4}{3x^2 - 2x + 4}\right)\right]$$

$$= \left[2 - \left(\frac{8 - 4 + 8 - 4}{12 - 4 + 4}\right)\right]$$

$$= 2 - \frac{8}{12} = 2 - \frac{2}{3} = \frac{4}{3}$$

Notes

9 TUE

Ques :- $x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}$ $x_0 = 0.5$

Obtain from N.R. method series converges at (means root of the series)

for bigger value of n $\rightarrow$ $x_{n+1} = x_n$

$$\alpha = \frac{\alpha}{2} + \frac{9}{8\alpha}$$

$$\alpha = \frac{4\alpha^2 + 9}{8\alpha}$$

$$8\alpha^2 = 4\alpha^2 + 9$$

$$4\alpha^2 = 9$$

$$\Rightarrow \alpha = \frac{3}{2}$$

10 WED

Ques :- $x_{n+1} = \frac{1}{2}\left[x_n + \frac{R}{x_n}\right]$

Can be used to compute ?

a) Square of R b) reciprocal of R (c) Square root of R d) log R.

$$\alpha = \frac{1}{2}\left[\alpha + \frac{R}{\alpha}\right].$$

$$\alpha = \sqrt{R}.$$

Notes

$f(x) = x^2 - R = 0$.

$$x_1 = x_0 - \left(\frac{x_0^2 - R}{2x_0}\right)$$

$$\left(\frac{2x_0^2 - x_0^2 + R}{2x_0}\right) = \left(\frac{x_0^2 + R}{2x_0}\right)$$

$$= \frac{1}{2}\left(x_0 + R/x_0\right)$$

$$\Rightarrow x_{n+1} = \frac{1}{2}\left(x_n + \frac{R}{x_n}\right)$$

Ques:- write iterative step to solve inverse of R.

$f(x) = Rx - 1 = 0$

$$\left[x - \left(\frac{Rx-1}{R}\right)\right] = \left(\frac{Rx - Rx + 1}{R}\right)$$

$= \frac{1}{R}$ (not a recursive step)

$$f(x) = \frac{1}{x} = R.$$

$$\frac{1}{x} - R = 0.$$

$$\therefore f(x) = \frac{1}{x} - R.$$

Notes

Sun	Mon	Tue	Wed	Thu	Fri	Sat
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

November 2012

13 SAT

$f'(x) = \frac{-1}{x^2}$.

$$= x - \frac{\{1/x - R\}}{-1/x^2}$$

$$= x + \left\{\frac{1}{x} - R\right\} x^2$$

$$= x + x - x^2 R$$
$$= 2x - x^2 R$$

Que :- write a recursive step to find inverse square root of R?

$$f(x) = \frac{1}{x^2} - R = 0.$$

14 SUN

$$f'(x) = \left(\frac{-2}{x^3}\right)$$

$$\Rightarrow x - \frac{f(x)}{f'(x)} = x - \frac{\left(\frac{1}{x^2} - R\right)}{\left(\frac{-2}{x^3}\right)}$$

$$= x + \left(\frac{1 - x^2 R}{2x^{2}}\right) x \, x^{2} x = x + \frac{1}{2}x - \frac{x^3 R}{2}$$

$$= (3x - x^3 R)\frac{1}{2}$$

$$= \frac{1}{2}x(3 - x^2 R)$$

Ques:- Write the recursive step to find p^{th} root of a number N.

$$f(x) = x^p - N = 0.$$

$$f'(x) = px^{p-1}$$

$$\Rightarrow x - \frac{f(x)}{f'(x)} = x - \left(\frac{x^p - N}{px^{p-1}}\right)$$

$$= \left(\frac{px^p - x^p + N}{px^{p-1}}\right)$$

$$= \left(\frac{(p-1)x^p + N}{px^{p-1}}\right)$$

Ques:- Iterative formula $\sqrt[3]{C}$ where $C > 0$ is

$$f(x) = x^3 - C = 0.$$

$$f'(x) = 3x^2.$$

$$\Rightarrow x - \frac{f(x)}{f'(x)} = \left[x - \left(\frac{x^3 - C}{3x^2}\right)\right]$$

$$= \left(\frac{2x^3 + C}{3x^2}\right)$$

$$\therefore x_{n+1} = \left(\frac{2x_n^3 + C}{3x_n^2}\right)$$

Notes

17 WED

Newton Raphson method is not applicable for

a) Constant function

b) For polynomial upto degree 1.

c) where $f'(x)$ vanishes.

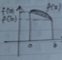
d) For non-polynomial function

	Guess value	I F	Replacement	order of Convergence	G
Bisection	2	$x_2 = \frac{x_0 + x_1}{2}$	$f_0 \cdot f_2 < 0$, $x_1 \leftarrow x_2$ or	1	low
Regula-falsi	2	$x_2 = \frac{f_1 x_0 - f_0 x_1}{f_1 - f_0}$	$f_1 \cdot f_2 < 0$, $x_0 \leftarrow x_2$	1	low
Secant	2		$x_0 \leftarrow x_1$, $x_1 \leftarrow x_2$	1.62	Not sure
Newton Raphson	1	$x_1 = x_0 - \frac{f(x)}{f'(x)}$		2	Not sure

18 THU

Trapezoidal method :-

$$\int_a^b f(x)\,dx$$



$h = \frac{b-a}{Ni}$

$Ni = \frac{b-a}{h}$

Area of Trapezium :-

$= \left(\frac{1}{2} \times \text{sum of } \parallel \text{ side} \times \text{height}\right)$

$= \frac{1}{2} \times [f(a) + f(b)]\,(b-a)$

$= \frac{b-a}{2}\,[f(a) + f(b)]$.

Notes

Simple Trapezoidal rule : (2-point System)

19 FRI

$b-a=h$

$\frac{h}{2}[f(0)+f(b)]$

or $\frac{h}{2}[f(0)+f(1)]$.

Compound Trapezoidal rule:-

$\frac{1}{2}(c-a)[f(a)+f(c)] + \frac{1}{2}(b-c)[f(c)+f(b)]$.

$\frac{h}{2}[f(a)+2f(c)+f(b)]$.

N-point System:-

$\frac{h}{2}[f(0)+2(f_1+f_2+f_3+\ldots f_{n-2})+f_{n-1}]$

Ques:-

20 SAT

The table below gives value of a function $f(x)$ obtained for value of x at interval of 0.25.

x	0	0.25	0.5	0.75	1.0
$f(x)$	1	0.9412	0.8	0.64	0.50

$\int_0^1 f(x)dx$.

$h=0.25$.

21 SUN $= \frac{0.25}{2}\left[1 + 2(0.9412 + 0.8 + 0.64) + 0.50\right]$

$= 0.7828.$

Ques:- A calculator has accuracy upto 8 places after decimal. $\int_0^{2\pi} \sin x \, dx$ when evaluated using this calculator by Trapezoidal method using 8 equal interval to 5 significant digit is ?

$h = \frac{\pi}{4}.$

22 MON $= \frac{\pi}{8}\left[\sin\frac{\pi}{4} + 2\left(\sin\frac{\pi}{2} + \sin\frac{3\pi}{4} + \sin\pi + \sin\frac{5\pi}{4} + \sin\frac{6\pi}{4} + \sin\frac{7\pi}{4}\right) + \sin 2\pi\right]$

$= \frac{\pi}{8}\left[0.70710 + 2(0.70710 + 1 + 0 - 0.70710 - 1 - 0.70710) + 0\right]$

$=$

September 2012	Sun	Mon	Tue	Wed	Thu	Fri	Sat
	30						1
	2	3	4	5	6	7	8
	9	10	11	12	13	14	15
	16	17	18	19	20	21	22
	23	24	25	26	27	28	29

A second degree polynomial $f(x)$ has valyee of 1, 4 and 15 at $x = 0, 1$ and 2 respectively. The $\int f(x)\,dx$ is to be estimated by applying the trapezoidal rule to this data. what is the error?

error = True value − Approximate value.

2^{nd} degree polynomial

$a_0x^2 + a_1x + a_2$

$x = 0 \to 1$, $x = 1 \to 4$, $x = 2 \to 15$.

$a_0x_0^2 + a_1x_0 + a_2 = 1$.

$a_2 = 1$.

$a_0 + a_1 + 1 = 4 \Rightarrow a_0 + a_1 = 3$.

$4a_0 + 2a_1 = 3 + 11$

$\Rightarrow 2a_0 + a_1 = \cancel{14}/2 = 7$

$2a_0 - a_0 = 14/2 - 3 = \cancel{3/2}\ 4$.

$\Rightarrow a_0 = \cancel{3/2}\ 4$.

$\cancel{\&}\ a_1 = -1$.

Notes

25 THU

$$\int_0^2 f(x)\,dx = \int_0^2 (4x^2 - x + 1)\,dx$$

$$= \left[\frac{4x^3}{3} - \frac{x^2}{2} + x\right]_0^2 = \frac{32}{3}.$$

$$\underset{0}{|}\xleftrightarrow{h}\underset{1}{|}\xleftrightarrow{h}\underset{2}{|}$$

$$\frac{h}{2}\left[f_0 + 2f(1) + f(2)\right]$$

$$= \frac{1}{2}\left[1 + 2\times4 + 15\right] = 12.$$

$$\text{error}: \frac{32}{3} - 12 = -\frac{4}{3}.$$

* Error in T. Method :-

26 FRI

$$T_E = -\frac{h^3}{12} \times \max\left(f''(\xi)\right)$$

(Simple Trapezoidal rule)

$$T_E = -\frac{h^3}{12} \times \max\left(f''(\xi)\right) \times Ni.$$

$$|T_E| = \frac{h^3}{12} \times Ni \times \max\left(f''(\xi)\right).$$

absolute error.

$$[a \le \xi \le b].$$

Notes

Ques :- The minimum number of equal length subinterval needed to approximate $\int_1^2 xe^x dx$ to an accuracy of atleast $\frac{1}{3} \times 10^{-6}$ using Trapezoidal rule is ?

Absolute error $= \frac{h^3}{12} \times N_i \times \max(f''(\xi))$

$f(x) = xe^x$

$f'(x) = e^x + xe^x$

$f''(x) = e^x + e^x + xe^x$

$= 2e^x + xe^x$

$|TE| = \left(\frac{1}{N_i}\right)^3 \times \frac{N_i}{12} \times \max(f''(\xi))$

$\max_{[1,2]} (f''(x)) = 2e^2 + 2e^2 = 4e^2$

$\frac{1}{3} \times 10^{-6} = \frac{1}{3} \times \frac{1}{N^2}$

$\Rightarrow N^2 = 10^6$

$\Rightarrow N = 1000$.

Notes

Ques:- The trapezoidal method to numerically obtain integral

29 MON $\int_a^b f(x)\,dx$ has an error, e bounded by

$\frac{b-a}{12} \times h^2 \times \max(f''(x))$ where $x \in$ a to b and h

is the width of the trapezoid. The minimum number of Trapezoid guaranteed to ensure $e < 10^{-4}$ in computing ln 7 using $f = \frac{1}{x}$.

$f(x) = \frac{1}{x}$, $f'(x) = \frac{1}{x^2}$, $f''(x) = \frac{2}{x^3}$

$\int_a^b \frac{1}{x}\,dx = [\ln x]_1^7 = \ln 7 - \ln 1 = \ln 7$

30 TUE $10^{-4} = \frac{6}{12} \times h^2 \times 2$

$h^2 = 10^{-4}$

$\Rightarrow h = 10^{-2}$

$\frac{6}{Ni} = 10^{-2}$ $\Rightarrow Ni = 600$.

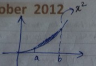
Ques:- If trapezoidal method is used to Compute $\int_0^1 x^2\,dx$ then value obtained is always.

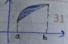
(a) Always greater than $\frac{1}{3}$

b) " less than $\frac{1}{3}$

c) " equal to $\frac{1}{3}$

d) greater than or less than $\frac{1}{3}$





Simpson Rule :-

$\frac{1}{3}$ Simpson Rule :- (odd number of points or even number of intervals)

$$\int_a^b f(x)\,dx = \frac{h}{3}[f(0) + 4f_1 + 2f_2 + 4f_3 + \ldots\ldots + f_n]$$

0 1 2 3 4

$$= \frac{h}{3}[f(0) + 4(f(1)) + 2(f(2)) + 4(f(3)) + f(4)]$$

x	0	0.25	0.5	0.75	1.0
$f(x)$	1	0.9412	0.8	0.64	0.50

$$= \frac{0.25}{3}[1 + 4\times0.9412 + 2\times0.8 + 4(0.64) + 4(0.5)]$$

$= 0.7854.$

Notes

1 THU

Ques:- Evaluate $\int_0^1 \frac{1}{1+x} dx$ using Trapezoidal rule, Simpson rule where $h = 0.5$.

$f(x) = \left(\frac{1}{1+x}\right)$

$f(0) = 1$
$f(0.5) = 1/1.5$
$f(1) = 1/2$

Trapezoidal:-

$= \frac{0.5}{2}\left[f(0) + 2(f(0.5)) + f(1) \right]$

$= 0.70834.$

Simpson:-

$= \frac{0.5}{3}\left[f(0) + 4f(0.5) + f(1) \right]$

2 FRI

$= 0.6945.$

Actual:- 0.6931.

Imp

→ Simpson is more accurate than Trapezoidal.

$T_E = \frac{-h^{(2+3)}}{90} \max f^{IV}(\xi_1)$

$|T_E| = \frac{h^5}{90}\left(\max f^{IV}(\xi_1) \right).$

Notes

→ Trapezoidal rule gives no error in integrating polynomial upto degree 1 (0,1)

3 SAT

→ Simpson rule gives no error for . . . ? degree 3 {0, 1, 2, 3}

Book: Knneth Rosen

3) Group Theory → NARSINGH DEO

Saumsouls → Proband stats.

1) Sequence and series
2) Log
3) Surds
4) Calculus.

4 SUN

5 MON

Propositional logic:-

→ Proposition
→ Logical Connectives
→ Implication and Bidirectional implication
→ well formed formula
→ Tautology, contingency and contradiction
→ Predicate Calculus.
→ Argument and its validity

Proposition:-

1) Today is Monday ✓
2) ~~Proposit~~ Gate Exam is on Feb-14. ✓
3) 2+2=4 ✓
4) 2+2=5 ✓
5) write it properly. X

6 TUE

6) what is your name? X
7) Oh my God! You got AIR-1 X.
8) x+2=5 X

A preposition is a declarative statement whose truth values are true or false but not both

Compound proposition:-
Proposition made up of one or more proposition along with logical Connectives.

Logical Connectives:- { ∨, ∧, ¬, }

∨ → Disjunction; ∧ → Conjunction; ¬ → NOT.

Notes

P: Jack went up to hill

Q: Jill went up to hill.

7 WED

Jack and Jill went up to the hill.

$P \wedge Q$.

1. Mark is poor but happy.
2. Mark is rich or unhappy.
3. Mark is neither rich nor happy.
4. Mark is poor or he is both rich and unhappy.

R : Mark is rich

H : Mark is happy.

8 TH

1) $\sim R \wedge H$

2) $R \vee \sim H$

3) $\sim R \wedge \sim H$

4) $\sim R \vee (R \wedge \sim H)$

(either, or) $\rightarrow$ $(P \vee Q)$

(neither, nor) $\cdot \sim$ (either, or)

1) $p \uparrow q \equiv \sim(p \wedge q)$

2) $p \downarrow q \equiv \sim(p \vee q)$

3) $p \,\bar{\vee}\, q \equiv (p \wedge (\sim q)) \vee ((\sim p) \wedge q)$

Notes

or $p \oplus q$.

9 FRI

p	q	$p\vee q$	$p\wedge q$	$\sim p$	$\sim q$	$p\uparrow q$	$p\downarrow q$	$p\bar{\vee} q$
T	T	T	T	F	F	F	F	F
T	F	T	F	F	T	T	F	T
F	T	T	F	T	F	T	F	T
F	F	F	F	T	T	T	T	F

Implication and Bidirectional :-

p implies q or $p \to q$.

10 SAT

P	Q	$P \to Q$ $(\bar{P}+Q)$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

$p \leftrightarrow q = p \odot q = pq + \bar{p}\bar{q}$

$= \sim(p \bar{\vee} q)$

1) If p then q
2) p implies q.
3) q if p
4) p only if q.
5) q unless ~p
6) p is sufficient for q.
7) q is necessary for p.
8) q whenever p.

11 SUN

$p \leftrightarrow q$.

1) If p then q and if q then p
2) p iff q
3) $(p \to q) \wedge (q \to p)$
4) p is sufficient for q and q is sufficient for P.
5) p is necessary and sufficient for q.

12 MON

Form and Implications :-

1) $p \to q$ (Implication)
2) $q \to p$ (converse)
3) $\sim p \to \sim q$ (Inverse)
4) $\sim q \to \sim p$ (Contrapositive)

1) I stay only if you go.
find converse of the following statement.

a) I stay if you go
b) If I stay then you go.

Notes

13 TUE

c) If you donot go then I donot stay.
d) If I donot stay then you go.

The crop will be destroyed if there is a flood.
c :- crop will be destroyed.
f :- There is a flood.

$f \to c$

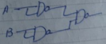
	Commutative	Associative
$\vee$	✓	✓
$\wedge$	✓	✓
$\uparrow$	✓	✗
$\downarrow$	✓	✗
$\overline{\vee}$	✓	✓
$\to$	✗	✗
$\leftrightarrow$	✓	✓

14 WED

$\uparrow$

$\vee$

$p \vee q = p + q$



Notes

October 2012

Sun	Mon	Tue	Wed	Thu	Fri	Sat
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

$\{\uparrow\}, \{\downarrow\}, \{(\vee, \sim)\}, \{(\wedge, \sim)\}, \{\rightarrow, \sim\}$

15 THU

minimal - $\{\uparrow\}, \{\downarrow\}$.

$\sim p \rightarrow q = p+q$

$\sim(p \rightarrow \sim q) = p \cdot q$.

→ PDNF (Principle disjunctive normal form) SOP

→ PCNF (Principle Conjunctive normal-form) POS.

1) $(P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$ (SOP)

$= (P \wedge Q \wedge (R \vee \bar{R})) \vee (\sim P \wedge R \wedge (Q \vee \bar{Q})) \vee (Q \wedge R \wedge (P \vee \bar{P}))$

$= (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \bar{R}) \vee (\sim P \wedge R \wedge Q) \vee (\sim P \wedge R \wedge \bar{Q}) \vee$

16 FRI

$\{Q \wedge R \wedge P\} \vee \{Q \wedge R \wedge \bar{P}\}$

ii) $(\sim p \rightarrow R) \wedge (Q \rightleftarrows P)$ (POS)

$= (p+r) \wedge ((\bar{Q}+P) \wedge (\bar{P}+Q))$

$= (p+r) \wedge (\bar{Q}\bar{P} + PQ)$

$= (p+r) \wedge (\bar{Q}+P) \wedge (\bar{P}+Q)$

$= (p+r+q\bar{q}) \wedge (\bar{Q}+p+r\bar{r}) \wedge (\bar{P}+Q+R\bar{R})$

$= (p+r+q) \wedge (p+r+\bar{q}) \wedge (\bar{Q}+p+r) \wedge (\bar{Q}+p+\bar{r}) \wedge$
$(P+Q+R) \wedge (\bar{P}+Q+\bar{R})$

Notes

17 SAT

The binary op $\Box$ is equivalent or defined as follows.

P	Q	P $\Box$ Q
T	T	T
T	F	T
F	T	F
F	F	T

Which of the following is equivalent to $p \vee q$.

a) $\sim Q \,\Box \sim P$ b) $P \,\Box\, \cdot Q$

c) $\sim P \,\Box\, Q$ d) $\sim P \,\Box \sim Q$

	$\bar{Q}$	Q
$\bar{P}$	1	
P	1	1

$P \Box Q = P + \bar{Q}$

$= Q \to P$.

18 SUN

a) $\sim Q \,\Box \sim P$

$\sim Q + P = P + \bar{Q}$

A	B	A o B
T	T	T
T	F	T
F	T	F
F	F	T

	$\bar{B}$	B
$\bar{A}$	1	0
A	1	1

$= P + \bar{Q}$

$= Q \to P$

Notes

a) $\sim A \odot B$ b) $\sim(A \odot \sim B)$
c) $\sim(\sim A \odot \sim B)$ d) $\sim(\sim A \odot B)$ 19 MON

$$\overline{(\sim B + \sim A)} = A \cdot B$$

Que:-

x	y	f(x,y)
0	0	0
0	1	0
1	0	1
1	1	1

f(x,y) ?

$$x\bar{y} + xy = x(\bar{y} + \bar{\bar{y}}) = x(1) = x$$

20 TUE

Que :- P and Q are two proposition which of the following logical expression are equivalent.

A) $P \vee \sim Q$
B) $\sim(\sim P \wedge Q)$
C) $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$
D) $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

1) only I and II 2) I, II and III 3) I, II, IV

4) All.

Notes

$A = B$

C) $PQ + P\bar{Q} + \bar{P}\bar{Q} = P + \bar{P}\bar{Q} = (P+\bar{P}).(P+\bar{Q}) = P+\bar{Q}$

D) $PQ + P\bar{Q} + \bar{P}Q = P + \bar{P}Q = P+Q$

$\therefore A = B = C$

Well formed formula (WFF)

→ A propositional variable (or symbol) is a WFF.

→ If p is a WFF then ¬p is also WFF.

→ If p and q are two WFF then $(p \vee q)$, $(p \wedge q)$ $(p \rightarrow q)$, $(p \leftrightarrow q)$ are also WFF.

22 Any finite combination of rule 1, 2 and 3 are in proper order is also a WFF.

WFF

Tautology — always True or identically True (Valid and satisfiable)

Contingency — T or F

Contradiction — always false

Notes

On solving the expression
1 → Tautology
0 → Contradiction
P → Contingency.

23 FRI

Which of the following is Tautology.

a) $(a \vee b) \to (b \wedge c)$
$\overline{(a+b)} + bc = \bar{a} \cdot \bar{b} + bc$

b) $(a \vee b) \to (\bar{b} + c)$ $= \overline{(a+b)} + \bar{b} + c$
$= \bar{a} \cdot \bar{b} + \bar{b} + c$

c) $(a \wedge b) \to (b \vee c)$ $= \overline{ab} + b + c = \bar{a} + \bar{b} + b + c = 1$

24 SAT

d) $(a \to b) \to (b \to c)$
$\overline{(\bar{a}+b)} + (\bar{b}+c) = \bar{\bar{a}} \cdot \bar{b} + \bar{b} + c$

e) $p \wedge (\sim p \vee q)$
$= p(\bar{p}+q) = pq \to$ Contingency.

Ques - Which of the following is tautology?

25 SUN

a) $(p \vee q) \to p$ - $\bar{P} \cdot \bar{Q} + p$.

b) $p \vee (Q \to P)$ = $p + (\bar{Q} + p)$

c) $p \vee (p \to q)$. $p + (\bar{p} + q)$

d) $p \to (p \to q)$ - $\bar{p} + \bar{p} + Q$ - $\bar{p} + Q$

Ques :-

$(P \to (Q \vee R)) \to ((P \wedge Q) \to R)$

a) satisfiable but not valid.

b) valid

c) a contradiction

26 MON

d) N.O.T.

$(\bar{P} + (Q + R)) \to (\overline{PQ} + R)$

$(\bar{P} + Q + R) \to (\bar{P} + \bar{Q} + R)$

$= \overline{(\bar{P} + Q + R)} + \bar{P} + \bar{Q} + R$

$= (P \cdot \bar{Q} \cdot \bar{R}) + \bar{P} + \bar{Q} + R$

$\bar{Q} + \bar{P} + R$.

Ques:- **November 2012**

October 2012	Sun	Mon	Tue	Wed	Thu	Fri	Sat
		1	2	3	4	5	6
	7	8	9	10	11	12	13
	14	15	16	17	18	19	20
	21	22	23	24	25	26	27
	28	29	30	31			

$P_1 : ((A \wedge B) \rightarrow C) \equiv ((A \rightarrow C) \wedge (B \rightarrow C))$

$P_2 : ((A \vee B) \rightarrow C) \equiv ((A \rightarrow C) \vee (B \rightarrow C))$ 27 TUE

Which of the following is true?

a) P_1 is tautology P_2 is not.
b) P_2 is and P_1 is not
c) P_1 is and P_2 is.
d) P_1 is not and P_2 is not.

L.HS

$P_1 = \overline{A \cdot B} + C = \bar{A} + \bar{B} + C$

$P_2 = \overline{A + B} + C = \bar{A}\bar{B} + C$

R.H.S

$(\bar{A} + C)(\bar{B} + C) = \bar{A}\bar{B} + \bar{A}C + C + C\bar{B}$ 28 WED

$= \bar{A}\bar{B} + C.$

R.H.S $(\bar{A} + C) + \bar{B} + C = \bar{A} + \bar{B} + C$

$(A + \bar{B} + C)(A\bar{B} + C) + (AB\bar{C})(\overline{A\bar{B} + C})$

Notes

If $P \equiv q$ if $P \leftrightarrow q$ is a tautology.

Ques:- Let P, Q, R be three atomic proposition
Let X denote $(P \vee Q) \rightarrow R$
Y , $(P \rightarrow R) \vee (Q \rightarrow R)$

a) $X \equiv Y$ b) $X \rightarrow Y$ c) $Y \rightarrow X$ d) $\neg Y \rightarrow X$

X: $\bar{P} \cdot \bar{Q} + R$

Y: $(\bar{P} + R) + (\bar{Q} + R)$ · $\bar{P} + \bar{Q} + R$

c) $Y \rightarrow X$

$= \overline{\bar{P} + \bar{Q} + R} + \bar{P}\bar{Q} + R$

$= (\bar{\bar{P}} . \bar{\bar{Q}} . \bar{R}) + \bar{P}\bar{Q} + R$

$= P.Q.\bar{R} + \bar{P}\bar{Q} + R$

$=$

d) $\bar{Y} + X = Y + X$

$= \bar{P} + \bar{Q} + R + \bar{P}\bar{Q} + R$

$= \bar{P} + R + \bar{Q}$

b) $(\overline{\bar{P}\bar{Q} + R}) + (\bar{P} + \bar{Q} + R)$

$= (\overline{\bar{P}\bar{Q}} . \bar{R}) + \bar{P} + \bar{Q} + R$

$= (P + Q)\bar{R} + \bar{P} + \bar{Q} + R =$

Notes

$= P\bar{R} + Q\bar{R} + \bar{P} + \bar{Q} + R$

$= \bar{P} + \bar{R} + QR + \bar{Q} + R$

$= 1$

Predicate Calculus

$x > 5$

variable is greather than 5

$\forall x\, P(x) \rightarrow$ False

$\exists x\, (P(x)) \rightarrow$ True.

$P(x) \rightarrow$ Predicate function

$P(x)$

$x = 10$ T

$x = 5$ F

$x = 100$ T.

Domain

$Q(x, y)$: $x + y = y + x$.

$\forall x \forall y\, Q(x, y)$ is true.

$\exists x \exists y\, Q(x, y)$ is true.

$\forall x\, P(x)$ universal quantifier

$\exists x\, P(x)$ existential quantifier.

Domain or universe of discourse

$P(x) = x^2 = +ve.$

$\forall x P(x)$: T

$\exists x P(x)$: T

Notes

3 MON

$\forall x P(x) \rightarrow \exists x P(x)$ } → True
$\exists x P(x) \rightarrow \forall x P(x)$ } → False

$D = \{1, 2, 3, \ldots\ldots, 10\}$

$P(x) : x^2 < 16$.
$\forall x P(x) \rightarrow$ False
$\exists x P(x) \rightarrow$ True.

$\forall x P(x) : P(a_1) \wedge P(a_2) \wedge P(a_3) \wedge \ldots\ldots \wedge P(a_n)$
$\exists x P(x) : P(a_1) \vee P(a_2) \vee P(a_3) \vee \ldots\ldots \vee P(a_n)$.

$\left.\begin{array}{l} \forall x \forall y \\ \exists x \exists y \\ \forall x \exists y \\ \exists x \forall y \end{array}\right\}$ possibilities.

4 TUE

$x, y, z \in$ Real number

$\Phi(x, y) : x + y = z$ {Sum of two real number is a real number}.
$\forall x \exists y \rightarrow$ inverse in Group theory.
$\exists x \forall y \rightarrow$ identity element.

Square of a -ve real number is positive.
Domain :- real number
$\forall x$ (If $x < 0$ then $x^2 > 0$)
$\forall x (x < 0 \rightarrow x^2 > 0)$.

Notes

All student of this class has studied Set Theory.

5 WED

Domain: 1. All the people of kanpur.
2. All the student of this class.
$S(x)$: x is a student of this class.
$St(x)$: x has studied set Theory.

1: $\forall x(S(x) \rightarrow St(x))$

2: $\forall x(St(x))$

Someone in this class has studied set theory.
1. $\exists x(S(x) \wedge St(x))$

2. $\exists x(St(x))$

6 THU

All, every, All :- $\forall(x)$
Some, there exist, atleast one, there are Some.

1. All lions are fierce.
2. Some lions don't drink Coffee
3. Some fierce creature drink coffee.
$l(x)$: x is a lion
$C(x)$: x drink coffee.
$f(x)$: x is fierce.

Notes

7 FRI

1) $\forall x(L(x) \rightarrow f(x))$
2) $\exists x(L(x) \wedge (\sim C(x)))$
3) $\exists x(f(x) \wedge C(x))$

Que :-

1) All graphs are connected.
2) Some graphs are not acyclic
3) Some connected graphs are cyclic

$G(x)$: x is a graph
$C(x)$: x is connected
$Cy(x)$: x is cyclic.

1) $\forall x(G(x) \rightarrow C(x))$
2) $\exists x(G(x) \wedge \sim Cy(x))$
3) $\exists x(G(x) \wedge C(x) \wedge Cy(x))$

8 SAT

Que :

1. No one is perfect
2. Not everyone is perfect.
3. Your all friends are perfect
4. None of your friends are perfect.

$P(x)$: x is perfect.
$F(x)$: x is your friend.

1. $\forall x(\sim P(x))$
2. $\exists x(\sim P(x))$
3. $\forall x(F(x) \rightarrow P(x))$
4. $\forall x\{F(x) \rightarrow \neg P(x)\}$
$\sim(\exists x(F(x) \wedge P(x)))$.

Notes

1. Tiger and lion are ferrocious.
2. Diamond and pearls are precious.
3. Gold and silver ornaments are precious.
4. Fast and car is dangerous

1. $\forall x \, (T(x) \vee L(x) \rightarrow F(x))$
2. $\forall x \, (D(x) \vee P(x) \rightarrow Pr(x))$
3. $\forall x \, (G(x) \vee S(x) \rightarrow Pr(x))$
4. $\forall x \, (F(x) \wedge C(x) \rightarrow Pr(x))$

Ques :-

1) All humming birds are richly coloured.
2) No large birds lives on honey
3) All birds that do not live on honey are dull in color.
4) Humming birds are small.
5) Not all birds live on honey.

$B(x)$: x is HB.

$C(x)$: x is richly coloured.

$H(x)$: x lives on honey.

$L(x)$: x is large

1) $\forall(x) \, (B(x) \rightarrow C(x))$
2) $\forall(x) \, (\sim(L(x)) \vee \sim H(x))$

   $\forall x \, (L(x) \rightarrow \neg H(x))$

   $\sim \exists x \, (L(x) \wedge (H(x)))$
3) $\forall x \, (\sim H(x) \rightarrow \neg C(x))$
4) $\forall x \, (B(x) \rightarrow \sim L(x))$
5) $\sim \forall(x) \, (L(x) \rightarrow H(x))$

   $\exists x \, (L(x) \wedge \sim H(x))$

Notes

11 TUE

1. Consider the following wff :-

I. $\neg \forall x P(x)$

II. $\neg \exists x P(x)$

III. $\neg \exists x (\sim P(x))$ I and IV

IV. $\exists x (\sim P(x))$

which of the following are equivalent.

2. Some real no are rational.

3. Not every graph are connected.

2) $\exists x(\text{Real}(x) \wedge \text{rational}(x))$

3) $\sim \forall x \{ G(x) \rightarrow C(x) \}$

$\exists x \{ \sim (\sim G(x)) \wedge \sim C(x) \}$

12 WED

(1) For every positive real number x and -ve real no y product of x and y is -ve real no.

$\forall x \forall y (x > 0 \& y < 0 \rightarrow xy < 0)$.

(2) Every real no except zero has multiplicative inverse

$\forall x \; (x \neq 0 \Rightarrow \exists y (xy = 1))$

(3) Sum of two +ve integer is +ve.

$\forall x \forall y (x > 0 \& y > 0 \rightarrow x + y > 0)$

Notes

→ If a person is female and his parents then this person is someone's mother. 13 THU

P(x): x is a ~~person~~ parent
F(x): x is female
M(x,y): x is mother of y.

$\forall x\ (F(x) \wedge P(x) \rightarrow \exists y\ (M(x,y)))$
$\forall x \exists y\ (P(x) \wedge F(x) \rightarrow M(x,y))$

Every teacher is liked by somestudents.

a) $\forall x\,(\text{teacher}(x) \rightarrow \exists y\,(\text{Student}(y) \rightarrow \text{likes}(y,x))$
b) $\forall x\,(\text{teacher}(x) \rightarrow \exists y\,(\text{Student}(y) \wedge \text{likes}(y,x)))$
c) $\exists y \forall x\,(\text{teacher}(x) \rightarrow (\text{Student}(y) \wedge \text{likes}(y,x)))$
d) $\forall x\,[\text{teacher}(x) \wedge \exists y\,(\text{Student}(y) \rightarrow \text{likes}(y,x))]$ 14 FRI

Ques:-
Let FSA and PDA be two predicate such that
fsa(x) - x is a finite automata
pda(y) :- y is pushdown automata.
Let equivalent be another predicate such that
equivalent(a,b): a and b are equivalent.
then which of the following logic statement represent each FSA has an equivalent PDA.

a) $\forall x\ (fsa(x) \rightarrow \exists y\ \{pda(y) \wedge eq(x,y)\})$
b) $\forall y\ (\exists x\ fsa(x) \rightarrow pda(y) \wedge eq(x,y))$
c) $\forall x \exists y\ (fsa(x) \wedge pda(y) \wedge eq(x,y))$
d) $\forall x \exists y\ (fsa(y) \wedge pda(x) \wedge eq(x,y))$

Ques :- Tiger and lion attack if they are hungry or threatened.

1) $\forall x\, [\text{tiger}(x) \wedge \text{lion}(x) \rightarrow \{\text{hungry}(x) \vee \text{threatened}(x) \rightarrow \text{attack}(x)\}\}$

2) $\forall x\, [\text{tiger}(x) \vee \text{lion}(x) \rightarrow \{\text{hungry} \vee \text{threatened} \rightarrow \text{attacks}(x)\}$

3) $\forall x\, [\text{tiger}(x) \vee \text{lion}(x) \rightarrow \{\text{attacks}(x) \rightarrow \text{hungry} \vee \text{threatened}(x)\}$

4) $\forall x\, [\text{tiger}(x) \vee \text{lion}(x) \rightarrow \{\text{hungry} \wedge \text{threatened} \rightarrow \text{attacks}\}]$

Ques: - Everyone has exactly one best friend.
B (x,y) : y is best friend of x.

No → Not a.
Not (______)
None (
None of your friends are perfect. → All of your friends are not perfect.

Notes

Arguement and its validity.
An arguement is valid if Conjunction of all the premises implication
Conclusion is a tautology.

P_1: If it will rain then I will carry umbrella.
P_2: It is raining.
∴ Q I am ~~not~~ carrying umbrella.

$$P_1 : p \to q$$
$$P_2 : \frac{p}{q}$$

$$(p \to q) \wedge p \to q$$
$$(\bar{p} + q) p \to q = \overline{qp} + q = \bar{q} + \bar{p} + q$$
$$= 1 + \bar{p}$$

Consider the following logical inferences.
1) If it rains then cricket match will not be played.
2) The cricket match was played.
∴ There was no rain.

$$(p \to \sim q) \wedge q \longrightarrow \sim p.$$
$$(\bar{p} + \sim q)\, q \to \sim p$$
$$\overline{(\bar{p} q)} + \sim p$$
$$\bar{\bar{p}} + \bar{q} + \sim p \quad - \quad 1 + \bar{q} = 1.$$

Notes

∴ Inference is true.

19 WED

1) If it rains then cricket match will not be played.
2) It did not rain
The cricket match was played.

$$(p \to q) \wedge (\sim p) \to q$$
$$(\bar{p} + \sim q)(\sim p) \to q$$
$$\sim q \sim p \to q$$
$$q + p + \bar{q} = q + p \quad \text{(contingency)}$$

$$\frac{p}{\therefore p \vee q} \qquad p \to p \vee q$$
$$\bar{p} + p \vee q = 1 + q = 1$$

20 THU Rules of inference:-

1) $\dfrac{p}{\therefore p \vee q}$ addition rule of inference $\{p \to p \vee q\}$

2) $\dfrac{q}{\therefore p \vee q}$ $\{q \to p \vee q\}$

3) $\dfrac{p \to q}{\sim q \to \sim p}$ Rule of contrapositive

4) $p \to q$, $\sim q$, $\overline{\sim p}$ Modus Tollens

5) $p \to q$, p, $\overline{\therefore q}$ Modus ponens

6) $p \vee q$, $\sim p$, $\overline{\therefore q}$ Disjunctive syllogism

7) $p \to q$, $q \to r$, $\overline{p \to r}$ Hypothetical syllogism

Notes

8) $p \vee q$
$\sim p \vee r$ Resolution
$\therefore q \vee r$

10) $p \to q$
$r \to s$ Destructive delimma
~~...~~
$\sim q \vee \sim s$
$\therefore \sim p \vee \sim r$

9) $p \to q$
$r \to s$ Constructive delimma
$p \vee r$
$\therefore q \vee s$

11) p
q (Conjunction)
$p \wedge q$

Ques :-
Prove that R is a valid inference from premises

$$\left.\begin{array}{l} P \to Q \\ Q \to R \end{array}\right\} \to (P \to R)$$

and P

$$\frac{P}{R}$$

Ques :- Inference :- $R \vee S$.
$C \vee D$, $(C \vee D) \to \sim H$, $\sim H \to (A \wedge \sim B)$
$(A \wedge \sim B) \to R \vee S$.

$(C \vee D) \to (A \wedge \sim B)$
$(A \wedge \sim B) \to (R \vee S)$
$(C \vee D) \to (R \vee S)$
$(C \vee D)$
$(R \vee S)$

Notes

23 SUN

1) Ques :- $R \wedge (P \vee Q)$ is a valid Conclusion from the premises.

$P \vee Q$, $Q \to R$, $P \to M$, $\neg M$.

2) Ques :-

Show that the hypothesis is it is not sunny this afternoon and it is colder than yesterday.
we will go to swimming only if it is sunny.
If we donot go swimming then we will take a Caneotrip.
If we take a caneotrip then we will be home by sunset.
This leads to Conclusion → we will be home by sunset.

24 MON

1)

$$\frac{\begin{array}{c} P \vee Q \\ \sim Q \vee R \end{array}}{P \vee R} \qquad \frac{\begin{array}{c} P \vee R \\ \sim P \vee M \end{array}}{R \vee M} \qquad \frac{\begin{array}{c} R \vee M \\ \sim M \end{array}}{R} \qquad \frac{\begin{array}{c} R \\ P \vee Q \end{array}}{R \wedge (P \vee Q)}$$

2)

$\sim S$

C

$Swi \to S$

$\sim Swi \to Can$

$Can \to Sun$

$$\frac{\begin{array}{c} \sim Swi \vee S \\ Swi \vee can \end{array}}{S \vee can} \qquad \frac{\begin{array}{c} can \vee S \\ \sim can \vee Sun \end{array}}{S \vee Sun}$$

$$\frac{\begin{array}{c} S \vee sun \\ \sim S \end{array}}{Sun}$$

→ we will be home by Sunset.

Notes

Graph Theory:-

→ Types of Graph
→ Degree of graph
→ Representation of Graph
→ isomorphism of Graph.
→ operation on Graph.
→ walk, path, cycle.
→ Euler and Hamiltonian Graph.
→ Connectness of Graph
→ ~~planing~~ planarity of Graph
→ Tree and Spanning tree.
→ Colouring of Graphs.
→ Independent set.
→ Matching Graph.

Graph $G=(V,E)$ is set of vertices and set of edges such that each edge incident with an unordered pair of vertices (v_i, v_j)

$V=\{v_1, v_2, \ldots, v_n\}$

$E=\{e_1, e_2, \ldots, e_n\}$

* min Graph:- 0 → Graph with single vertex and no edge.
V = non empty.
E = can be empty.

Notes

27 THU

* Null Graph –

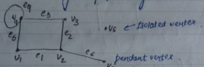
→ Graph with no edge.

* Empty Graph –

→ A Graph with no vertex and edge.

$V = \phi$ and $E = \phi$.

~~Inser~~ Intersection of two Graphs may be null.



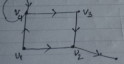
28 FRI

Self loop – an edge whose initial and final vertex are same.

pendant vertex :- a vertex associated with a single edge

isolated vertex :- a vertex associated with no edge.

Directed Graph :-



Notes

November 2012

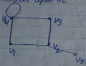
Sun	Mon	Tue	Wed	Thu	Fri	Sat
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

29 SAT

A graph $G = (V, E)$ is a set of vertices and set of edges and a mapping that maps each edge with an ordered pair of vertices (v_i, v_j)

→ Indegree
→ outdegree
→ incident edge
→ outgoing edge.

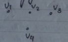
Degree of a vertex :-
no. of edge incident upon it.



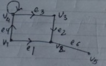
$d(v_1) = 2$
$d(v_2) = 3$
$d(v_3) = 2$
$d(v_4) = 4$
$d(v_5) = 1$

30 SUN

Null Graph of four vertices



$d(v_1) = d(v_2) = d(v_3) = d(v_4) = 0$



$in(v_1) = 0$, $out(v_1) = 2$
$in(v_4) = 2$, $out(v_4) = 2$

* $\sum d(v_i) = 2 \times e$ → Hand shaking theorem.

Notes

→ Degree sum of each vertex in an undirected Graph is even. → True.

$\sum d^-(v_i) = \sum d^+(v_i) = e$.

→ Number of vertices of odd degree is always even

$\sum d(v_i) = \sum_{\text{even}} d(v_i) + \sum_{\text{odd}} d(v_i)$

$= 2e$
(even)

$\sum_{\text{even}} d(v_i) =$ always even.

$\therefore$ $\sum_{\text{odd}} d(v_i)$ must be even.

$\therefore$ number of vertices of odd degree is always even.

2, 2, 3, 3, 5 → NOT POSSIBLE.

Regular Graph –
a Graph for which degree of each vertex is equal.

2-Regular Graph.

degree.

K – Regular Graph
K is degree of each vertex.

Notes

Find number of edge in a K-regular graph with n vertices.

$$n \times K = 2e$$

$$\Rightarrow e = \frac{nK}{2}$$

→ K-Regular Graph can never be of odd order (K is odd) → True.

Order(G) :- no. of vertices in a graph.

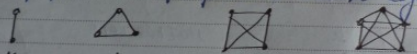
Size(G) :- no. of edges in a graph.

$$K \times n = 2e$$

$$odd \times odd = odd \quad \times \quad (\because \text{Not possible})$$

Complete Graph :- A simple Graph with max no. of edges or a (n-1) regular Graph with n vertices.

Simple Graph → no self loop and no multiple edges.



K_2 K_3 K_4 K_5.

2-Reg Graph with 4 vertices.

* All Complete Graphs are Regular however Converse may not be true.

Number of edges in a Complete graph $= \frac{n(n-1)}{2}$.

min - degree :- $\delta(G)$

max - degree :- $\Delta(G) = (n-1)$

Average degree → $\left(\frac{\text{Total Sum of degree of each vertex}}{\text{Total No. of vertices}} \right)$

$$= \frac{2e}{n}$$

$$\delta(G) \leq \frac{2e}{n} \leq \Delta(G)$$