Reactive Policies with Planning for Action Languages

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A Proof Sketches

Theorem 1 (soundness of Reach). *Let $\mathcal{T}_h{=}\langle \widehat{S}, \widehat{S}_0, G_B, \mathcal{B}, \Phi_{\mathcal{B}}\rangle$ be a transition system w.r.t. a classification function h. Checking whether every transition found by the policy execution function $\Phi_{\mathcal{B}}$ induced by a given implementation Reach is correct is in Π^p_3.*

Proof (Sketch). According to Definition 6, every transition from a state $\hat{s}$ to some state $\hat{s}'$ corresponds to some plan σ returned by $\mathit{Reach}(\hat{s}, g_B)$. Thus first one needs to check whether each plan $\sigma = \langle a_1, a_2, \dots, a_n\rangle$ returned by Reach given some $\hat{s}$ and g_B is correct. For that, we need to check two conditions on the corresponding trajectories of the plan: (i) for all partial trajectories $\hat{s}_0, \hat{s}_1, \dots, \hat{s}_{i-1}$ it holds that for the upcoming action a_i from the plan σ, $\hat{\Phi}(\hat{s}_{i-1}, a_i) \neq \emptyset$ (i.e., the action is applicable). (ii) for all trajectories $\hat{s}_0, \hat{s}_1, \dots, \hat{s}_n$, $\hat{s}_n \models g_B$. Checking whether these conditions hold is in Π^p_2.

Thus, to decide whether for some state $\hat{s}$ and target g_B the function $\Phi_B(\hat{s}, g_B)$ does not work correctly, we can guess $\hat{s}$ (resp. $s \in \hat{s}$), g_B, a plan σ, and verify that $\sigma \in \mathit{Reach}(\hat{s}, g_B)$ and that σ is not correct. As we can do the verification with an oracle for Σ^p_2 in polynomial time, correctness can be refuted in Σ^p_3; thus the problem is in Π^p_3.

Theorem 2 (completeness of Reach). *Let $\mathcal{T}_h = \langle \widehat{S}, \widehat{S}_0, G_B, \mathcal{B}, \Phi_{\mathcal{B}}\rangle$ be a transition system w.r.t. a classification function h. Deciding whether for a given implementation Reach, Φ_B fulfills $\hat{s}' \in \Phi_B(\hat{s})$ whenever a short conformant plan from $\hat{s}$ to some $g_B \in \mathcal{B}(\hat{s})$ exists and $\hat{s}'$ is the resulting state after the execution of the plan in T_h, is in Π^p_4.*

Proof (Sketch). For a counterexample, we can guess some $\hat{s}$ and $\hat{s}'$ (resp. $s \in \hat{s}$, $s' \in \hat{s}'$) and some short plan σ and verify that (i) σ is a valid conformant plan in $\mathcal{T}_h$ to reach $\hat{s}'$ from $\hat{s}$, and (ii) that a target g_B exists such that $\mathit{Reach}(\hat{s}, g_B)$ produces some output. We can verify (i) using a Π^p_2 oracle to check that σ is a conformant plan, and we can verify (ii) using a Π^p_3 oracle (for all guesses of targets g_B and short plans σ', either g_B is not a target for $\hat{s}$ or σ' is not produced by $\mathit{Reach}(\hat{s}, g_B)$). This establishes membership in Π^p_4.

Theorem 3. *The problem of determining whether the policy works is in* PSPACE.

Proof (Sketch). One needs to look at all runs $\hat{s}_0, \hat{s}_1, \dots$ from every initial state $\hat{s}_0$ in the equalized transition system and check whether each such run has some state $\hat{s}_j$ that satisfies the main goal g_∞. Given that states have a representation in terms of fluent or state variables, there are at most exponentially many different states. Thus to find a counterexample, a run of at most exponential length in which g_∞ is not satisfied is sufficient. Such a run can be nondeterministically built in polynomial space; as $\mathsf{NPSPACE} = \mathsf{PSPACE}$, the result follows.

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Theorem 4. *Let $\mathcal{T}_h = \langle \widehat{S}, \widehat{S}_0, G_B, \mathcal{B}, \Phi_\mathcal{B} \rangle$ be a transition system w.r.t. a classification function h. Let $\hat{\Phi}$ be the transition function that the policy execution function $\Phi_\mathcal{B}$ is based on. The problem of checking whether $\hat{\Phi}$ is proper is in Π_2^p.*

Proof (Sketch). As a counterexample, one needs to guess $\hat{s}, a$, $\hat{s}' \in \hat{\Phi}(\hat{s}, a)$ and $s' \in \hat{s}'$ such that no $s \in \hat{s}$ has $s' \in \Phi(s, a)$.

Proposition 1 (soundness). *Let $\mathcal{T}_h = \langle \widehat{S}, \widehat{S}_0, G_B, \mathcal{B}, \Phi_\mathcal{B} \rangle$ be a transition system w.r.t. a classification function h. Let $\hat{s}_1, \hat{s}_2 \in \widehat{S}$ be equalized states that are reachable from some initial states, and $\hat{s}_2 \in \Phi_B(\hat{s}_1)$. For any concrete state $s_2 \in \hat{s}_2$, assuming (2), there is a concrete state $s_1 \in \hat{s}_1$ such that $s_1 \rightarrow^\sigma s_2$ for some action sequence σ, in $\mathcal{T}$.*

Proof. For equalized states $\hat{s}_1, \hat{s}_2$, having $\hat{s}_2 \in \Phi_B(\hat{s}_1)$ means that $\hat{s}_2$ satisfies a target condition that is determined at $\hat{s}_1$, and is reachable via executing some plan σ. Assuming that (2) holds, we can apply backwards tracking from any state $s_2 \in \hat{s}_2$ following the transitions Φ corresponding to the actions in the plan σ backwards. In the end, we can find a concrete state $s_1 \in \hat{s}_1$ from which one can reach the state $s_2 \in \hat{s}_2$ by applying the plan σ in the original transition system.

B Reachability of States in the Equalized Transition System

A state $\hat{s}$ is *reachable* from an initial state in the equalized transition system if and only if $s \in \mathcal{R}_i$ for some $i \in \mathbb{N}$ where $\mathcal{R}_i$ is defined as follows.

$$\mathcal{R}_0 = \widehat{S}_0,\ \mathcal{R}_{i+1} = \textstyle\bigcup_{\hat{s} \in \mathcal{R}_i} \Phi_B(\hat{s}),\ i \geq 1, \text{ and } \mathcal{R}^\infty = \textstyle\bigcup_{i \geq 0} \mathcal{R}_i.$$

Under the assumptions that apply to the previous results, we can state the following.

Theorem 5. *The problem of determining whether a state in an equalized transition system is reachable is in* PSPACE.

The notions of soundness and completeness of an outsourced planning function $Reach$ could be restricted to reachable states; however, this, would not change the cost of testing these properties in general (assuming that $\hat{s} \in \mathcal{R}$ is decidable with sufficiently low complexity).